ON HYPERFILTERS IN TERNARY SEMIHYPERRING

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ABSTRACT:
Here, we introduce the concept of irreducible hyper ideals in ternary semi hyper rings and investigate some of the basic properties on irreducible hyper ideals. We also investigate the notion of hyper filters in the ternary semi hyper rings and give some characterizations of hyper filters in ternary semi hyper rings. And also some relationships between hyper filters, prime hyper ideals and in ternary semi hyper rings are also to be considered.

Key words: “Semi hyper ring”, “Ternary semi hyper ring”, “hyper ideal”, “prime hyper ideal”, “semi prime hyper ideals”, “Hyper filter”.

Mathematical Subject Classification: 20N20, 16Y99.

I. INTRODUCTION:
In 8th congress of Scandinavian Mathematicians, Hyper structure theory was introduced in the year 1934. In [5] investigated the notion of hyper group as generalization of groups. In [2] the authors are studied about binary relations in ternary hyper groups and characterized them. The main theme of this paper is to studied about hyper filters in ternary semi hyper rings.

II. PRELIMINARIES:
Let a non-empty set H and ◦ : H × H → ℘∗(H) be a hyper operation, and where ℘∗(H) be a power set of H. Then (H, ◦) is nothing but “hyper groupoid” if for any (∅ ≠)r, (∅ ≠)s ⊆ H, r, s ⊆ H we have 

\[ r ◦ s = ∪ \{ r ◦ u | u ∈ s \}. \]

A ternary hyper grouped is known as the pair (H, [ ] ) if (∅ ≠)H₁, (∅ ≠)H₂, (∅ ≠)H₃ ⊆ H then we define 

\[ [H₁,H₂,H₃] = ∪ \{ h₁h₂h₃ | h₁,h₂,h₃ ∈ H \}. \]

A ∅≠H is known as “ternary semi hyper ring” if ∀ h₁,h₂,h₃,h₄,h₅ ∈ H as well as (H, ⊕) is a “commutative semi hyper group”. For more preliminaries go through the references. Though out this paper we consider as “ternary semihyper ring” as TSHR, “Left hyper filter, lateral hyper filter as well as right hyper filter, hyper filters” as LHF, MHF, RHF, HF respectively.

III. HYPER FILTERS
Def 3.1: Let H be a TSHR, F be a sub TSHR of H is known as a LHF(MHF, RHF) of H if f₁, f₂, f₃ ∈ H, [f₁f₂f₃] ∈ F ⇒ f₁ ∈ F, f₂ ∈ F, f₃ ∈ F.

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Example 3.2: Let \( H = \{ a, t, p, d \} \) along with “hyper addition” as \( s \oplus u = \{ \ t \in H \ / \ \min \{ s, u \} \leq t \leq \max \{ s, u \} \} \), “ternary multiplication” as

\[
[suz] = \begin{cases} 
    a & \text{otherwise} \\
    t & \text{if } s = u = z = t \\
    p & \text{if } s = u = z = p \\
    d & \text{if } s = u = z = d 
\end{cases}
\]

Then \( H \) is a TSHR and \( \{ a, t, p, d \}, \{ t \}, \{ p \}, \{ d \} \) are all the HF's of \( H \).

Th 3.3: Let \( H \) be a TSHR, then the non empty intersection of a family of HF's of a TSHR \( H \) is also HF of \( H \).

Proof: Let \( \{ G_{\alpha} \}_{\alpha \in \Delta} \) be a family of HF's of \( H \), let

\[
G = \bigcap_{\alpha \in \Delta} G_{\alpha}.
\]

Let \( s, u, c \in H, [suc] \in G \). Now \( [suc] \in G \Rightarrow [suc] \in \bigcap_{\alpha \in \Delta} G_{\alpha} \Rightarrow [suc] \in G_{\alpha} \).

\( [suc] \in G_{\alpha}, G_{\alpha} \) is a HF of \( H \) \( \Rightarrow s, u, c \in G_{\alpha} \).

\( \Rightarrow s, u, c \in \bigcap_{\alpha \in \Delta} G_{\alpha} \Rightarrow s, u, c \in G. \Rightarrow G \) is a HF of \( H \).

Note 3.4: In general, let \( H \) be a TSHR, then the union of two HF's is not a HF.

Example 3.5: As above mentioned in example 3.2, \( H \) is a ternary semi hyper ring and \( \{ b \}, \{ c \} \) are hyper filters, but \( \{ b \} \cup \{ c \} \) is not a hyper filter of \( H \) because \( bcc = a \notin \{ b \} \cup \{ c \} \).

Now we have to prove \( \iff \) condition for a nonempty subset to be a HF in a TSHR.

Th 3.6: Let \( H \) be a TSHR, \( a \emptyset \neq P \subseteq H \) is a HF \( \iff H \setminus P \) is a “completely prime hyper ideal” of \( H \) or empty.

Proof: Let \( H \setminus P \neq \emptyset, s, u, z \in H \setminus P, [suz] \notin H \setminus P, \) then \( [suz] \in P \). \( \because P \) is a HF \( \Rightarrow s, u, z \in F \).

It is a contradiction. Thus \( [suz] \in H \setminus P \), and so \( (H \setminus P)(H \setminus P) \subseteq H \setminus P \).

Therefore \( H \setminus P \) is a “hyper ideal”.

Next we have to show that \( H \setminus P \) is a “completely prime”.

Let \( [suz] \in H \setminus P \) for \( s, u, z \in H \setminus P \).

Suppose that \( s, u, z \notin H \setminus P \).

Then \( s, u, z \in P \). Since \( P \) is a “hyper sub semi ring” of \( H[suz] \in P \). It is absurd.

Thus \( s \in H \setminus P \) or \( u \in H \setminus P \) or \( z \in H \setminus P \) implies that \( H \setminus P \) is “completely prime” \( \Rightarrow H \setminus P \) is a “completely prime hyper ideal” of \( H \).

Conversely, \( H \setminus P \) is a “completely prime hyper ideal” of \( H \) or empty.

If \( H \setminus P = \emptyset \), then \( P = H \). Thus \( P \) is a HF of \( H \).
Let $H \setminus P$ be a “completely prime hyper ideal” of $H$.

if $\forall s, u, z \in P$, $[suz] \notin P$. Then $suz \in H \setminus P$.

Since $H \setminus P$ is “completely prime”, $s \in H \setminus P$ or $u \in H \setminus P$ or $z \in H \setminus P$. It is absurd.

Thus $[suz] \in P$ \Rightarrow $P$ is a sub TSHR of $H$.

Let $s, u, z \in H$, $[suz] \subseteq H \setminus P$. If $s, u, z \notin P$, then $s, u, z \in H \setminus P$.

Since $H \setminus P$ is a “completely prime hyperideal” of $H$, $[suz] \subseteq (H \setminus P)(H \setminus P)(H \setminus P) \subseteq H \setminus P$.

It is absurd. Thus $s, u, z \in P$.

Therefore $P$ is a HF of $H$.

Th 3.7: Let $H$ be a TSHR then every “completely prime ideal” of a TSHR $H$ is a “prime hyper ideal” of $H$.

Corollary 3.8: Let $H$ be a TSHR. If $G$ is a HF of $H$, then $H \setminus G$ is a “prime hyper ideal” of $H$ or empty.

Proof: \[ \because \] $G$ is a HF of $H$. By th 3.6, $H \setminus G$ is a “completely prime hyperideal” of $H$ or empty.

Let $s, u, z \in H$ and $<s><u><z> \subseteq H \setminus G$. Then $[suz] \subseteq H \setminus G$.

\[ \because \] $H \setminus G$ is a “completely prime”, either $s \in H \setminus G$ or $u \in H \setminus G$ or $z \in H \setminus G$.

\[ \because \] $H \setminus G$ is a “prime hyper ideal” of $H$.

Th 3.9: In a commutative ternary semi hyper ring a hyper ideal is completely prime if and only if it is prime hyper ideal.

Corollary 3.10: Let $H$ be a TSHR, $\emptyset \neq G \subseteq H$ is a HF iff $H \setminus G$ is a “prime hyper ideal” of $H$ or empty.

Th 3.11: Let $H$ be a TSHR, then it don’t contain proper HF if and only if $H$ don’t proper completely prime hyper ideals.

Proof: Let us assume that the ternary semi hyper ring $H$ don’t contains proper HF. Let $C$ be a “completely prime hyper ideal” of $H$, $C \subseteq H$. Then $\emptyset \neq H \setminus C \subseteq H$ and $H \setminus C = C$ is a “completely prime hyper ideal” of $H$. \[ \because \] $H \setminus C$ is the complement of $C$ to $H$, by th 3.6, $H \setminus C$ is a HF of $H$. Then $H \setminus C = H \Rightarrow C = \emptyset$. It is absurd. \[ \because \] $H$ don’t contain “proper completely prime hyper ideals”.

Conversely, assume that $H$ do’ not contain proper “completely prime hyper ideals”. Let $C$ be a HF of $H$, $C \subseteq H$. \[ \because \] $H \setminus C \neq \emptyset$, by th 3.6, $H \setminus C$ is a “completely prime hyperideal” of $H$. Then $H \setminus C = H \Rightarrow C = \emptyset$. Which is a contradiction. \[ \because \] $H$ don’t contain proper HF.

Th 3.12: A TSHR $H$ and $P$ be a “proper hyper ideal” of $H$, then $P$ is a “prime hyper ideal” \[ \iff \] $H \setminus P$ is an m-system.

Th 3.13: Every HF $C$ of a TSHR $H$ is a “m-system” of $H$.

Proof: Let us assume $C$ is a HF of a TSHR $H$. By corollary 3.8, $H \setminus C$ is a “prime hyper ideal” of $H$. By th 3.12, $H \setminus (H \setminus C) = C$ is a “m-system” of $H$ or empty.

Th 3.14: Let $H$ be a TSHR, then Every “completely prime hyper ideal” of a TSHR $H$ is a “completely semi prime hyper ideal” of $H$.

Th 3.15: Let $H$ be a TSHR. If $G$ is a HF, then $H \setminus G$ is a “completely semi prime hyper ideal” of $H$.

Th 3.16: Every “completely prime hyperideal” of a TSHR $H$ is a “completely semi prime hyper ideal” of $H$.
Th 3.17: Let H is a TSHR. If G is a HF, then H\G is a “semi prime hyper ideal” of H.

Proof: Suppose that G is a HF of a TSHR H. By th 3.6, H\G is a “completely prime hyper ideal” of H. By th 3.4, H\G is a “completely semi prime hyper ideal” of H. By 3.16, H\G is a “semi prime hyper ideal” of H.

Def 3.18: Let H be a TSHR, ∅ ≠ G ⊆ H. The HF of H generated by G is the “smallest HF of H containing G, and it is denoted by N(G).

Th 3.19: The HF of a TSHR H generated by a non empty subset P which is the intersection of all HFs of H containing P.

Def 3.20: A HF G of a TSHR H is said to be a principal HF provided G is a HF which is generated by {a} for some a ∈ H. It is denoted by N(a).

Example 3.21: As in the above said example 3.2, H is a TSHR, N(a) = {a, t, p}, N(t) = {t}, N(p) = {p} & N(d) = {d} all are the principal HF of the TSHR H.

Corollary 3.22: Let H is a TSHR, p ∈ H. Then N(p) is the least HF of H containing {p}.

Note 3.23: For every x ∈ H, all the intersection of hyper filters containing {x} is again a hyper filter and thus the least hyper filter containing {x}.

Th 3.24: If N(b) is contained in N(a), then N(a)\N(b), if it is nonempty, is a “completely prime hyperideal” of N(a).

Proof: BY th 3.11, therefore N(a)\N(b) is a “completely prime hyperideal” of N(a).

Lemma 3.25: Let p, q ∈ H and q ∈ N(p), then N(q) ⊆ N(p).

Proof: From the definition of the principal hyper filter, it is clear.

Corollary 3.26: Let p, q ∈ H, p ≤ q then N(q) ⊆ N(p).

Proof: From the given condition p ≤ q then it is clear that q ∈ N(p). By lemma 3.25, we have N(q) ⊆ N(p).

IV. “ON IRREDUCIBLE HYPER IDEALS IN TERNARY SEMI HYPER RINGS”

The notion of “irreducible hyper ideals” in TSHRs and characterized them.

Def 4.1: Let H be a TSHR and a “proper hyper ideal” A of H is said to be “irreducible” if the “hyper ideals” T and S of H ∋ T ∩ S = A ⇒ T = A or S = A.

Def 4.2: Let H be a TSHR and a “proper hyperideal” A of H is said to be “strongly irreducible” if “hyper ideals” T and S of H ∋ T ∩ S ⊆ A ⇒ T ⊆ A or S ⊆ A.

Th 4.3: If L is a “hyper ideal” of H and a is a non zero element of T ∋ a ∉ L, then ∃ an “irreducible hyper ideal” P of T ⊇ L ⊆ P, a ∉ P.

Th 4.4: Any “proper hyper ideal” of H is the “intersection of irreducible hyperideal” of H which contained in it.

Proof: Assume that L be any “proper hyper ideal” of H and \{B_a\}_a∈Δ be a family of “irreducible hyper ideals” of H which contain L, here Δ represents the indexed set. Then clearly L ⊆ \bigcap_{a∈Δ} B_a. To show \bigcap_{a∈Δ} B_a ⊆ L. If \bigcap_{a∈Δ} B_a ⊆ L implies that ∃ a∈Δ \∩ B_a ∉ a ∉ L. Then by the th 4.3, ∃ an “irreducible hyper ideal” P ∋ L ⊆ P, a ∉ P. This
indicates the existence of “irreducible hyper ideal” $P \ni a \notin P$, $L \subseteq P$. ∴ $a \notin L$. Hence, by the contra
positive method $\bigcap_{a \in \Delta} B_a \subseteq L$. ∴ $\bigcap_{a \in \Delta} B_a = L$.

Th 4.5: Every “strongly irreducible” and “semi prime hyper ideal” is to be a “prime hyper ideal” of $H$.

Proof: Let $P$ be a “semi prime and strongly irreducible and a hyper ideal” of $T$. let us take any of the “hyper ideals” $E$, $S$ and $Z$ of $T$, $[E\cap S \cap Z] \subseteq P$. $E \cap S \cap Z$ is a “hyper ideal” of $H$. ∴ $(E \cap S \cap Z) \cap (E \cap S \cap Z) \cap (E \cap S \cap Z) \subseteq [E\cap S \cap Z] \subseteq P$. ∴ $P$ is a “semi prime hyper ideal”, $E \cap S \subseteq P$ or $S \subseteq P$ or $Z \subseteq P$. Here, $P$ is a “strongly irreducible hyper ideal”. ∴ $P$ is a “prime hyper ideal” of $H$.

Th 4.6: Any “proper hyper ideal” of $H$ is the “intersection of irreducible hyper ideal” of $H$ which contains it.

Proof: Assume that $R$ be any “proper hyper ideal” of $H$ and $\{X_I|I \in \Delta\}$ be a “family of irreducible hyper ideals” of $H$ which contains $R$, here $\Delta$ represents the indexed set. ∴ clearly $R \subseteq \bigcap I_X I$. To show that $\bigcap I_X I \subseteq R$. Let $\bigcap I_X I \ni X_I \ni a \notin R$. Then by above th 4.3, $\exists$ an “irreducible hyper ideal” $P \ni R \subseteq P$, $a \notin P$. This develop the existence of “irreducible hyper ideal” $P \ni a \notin P$, $R \subseteq P$. ∴ $a \notin \bigcap I_X I$ for every $a \notin R$. Hence, by the contra positive method $\bigcap I_X I \ni R$. ∴ $\bigcap I_X I = R$.

V. CONCLUSION:

The “ternary semi hyperring” is a universalisation of the concepts of a “semiring”, a “semi hyper ring”. Since, the notions of “hyper filters”, “irreducible hyper ideals”; in a “ternary semi hyperring” are introduced, and several examples given and characterized them.

Acknowledgements:

The authors express their whole hearted gratitude to the referees for useful and necessary comments and suggestions that improved the paper.

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