EVALUATES OF CROSS-SECTION DATA OF PROTON INDUCED REACTION AT HIGH ENERGY SCATTERING

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ABSTRACT

The interaction of zirconium cross-section with proton scattering is calculated using eikonal approximation operating at high energies (with small scattering angle). Eikonal approximation is based on the optical potential obtained through the folding integration of the material density used and the nucleon-nucleon transmission capacity. This new method of approximation increases the computational efficiency by three times the predicted speed of the elastic and inelastic cross-sectional dispersion and the interaction cross-sections.

I. INTRODUCTION

Experiments using rare isotopes are currently underway or expected, and they promise to advance our knowledge of nuclei and their importance in nuclear physics[1]. Nuclear reaction theory is needed for both interpreting the data and determining the experiments that are required[2][3]. The use of the eikonal approximation has long been regarded as an appealing method for simplifying calculations for medium and high energies[4][5]. The eikonal approximation has a relationship with the optical potential which depends on the density of the material and the transmission amplitude of the nucleons. Whereas, the nucleus is a complex of particles whose primary components consist of protons and neutrons[6]. Several researchers have done measurements that are extensive and precise of observables differential cross sections for protons’ elastic scattering from nuclei with a wide spectrum of proton energies[7].

Any explanation of the mechanism of nuclear reaction requires finding a solution to the problem of the many-body between the projectile and target nucleons. The interactions between the pairs groups of nucleons can be studied by using some simple models such as the optical potential or the double folding potential. In general, optical potential model is widely used to demonstrate elastic scattering through the use of empirical parameters, which should display the same behaviors of the interacting system by studying the differential cross-section which is the real and imaginary scattering amplitudes of the optical potential. To study the nuclear reaction for excited ⁹⁰Zr nuclide in the framework of the coupled-channels concerning performing the calculations of differential cross-section the p + ⁹⁰Zr interaction scattering, because it is expected that elastic scattering of protons will have an important experience in obtaining information about nuclei, especially stable ones, as in electron scattering. The suitable approximation which is the eikonal approximation that provides a simplified interpretation.

II. THEORY MODELS AND CALCULATION PARAMETERS

2.1 Optical Potential parameter of proton

An optical potential model is a complex potential used to describe the scattering interaction that used in approaches calculations described by compound two parts, the first is the deep potential and attraction parts of real potential and the second is represented by Saxon-Wood functions for weak and absorption parts of imaginary potential according to the equation [8]:

\[ U_{opt} = -V_s f(r, R_s, a_s) - iW_s (r, R_i, a_i) + 4a_d W_D \frac{d}{dr} f(r, R_i, a_i) \]  \( (1) \)

Where \( V_s \) and \( iW_s \) are the real and imaginary parts (that consists of a surface and a volume), \( (4a_d) \) is the surface shape factor, and \( f \) is the Saxon-Wood form factor given as
Microscopic optical potential approach has a real part that obtained by adopted a nucleon-nucleon effective interaction, while the macroscopic description does not treat the nucleus as a whole system consist of nucleons, then the interaction between the projectile and a target can be described in terms of a mean potential [9][10]. The imaginary potential in the microscopic description, there is no difference between surface and volume components [11]. In comparison to the actual central part of the optical potential, the microscopic imaginary part has significant qualitative differences as compared to phenomenology [12].

Table (1) shows the parameters of proton optical potential that chosen properly with \( r_c = 1.032 \) fm and \( V_V, W_W, W_s, V_{so}, \) are in (MeV) , \( r_V, r_W, r_{WS}, r_{SO}, r_c \), are in (fm), \( a_v, a_w, a_s, a_{so} \), in (fm).

<table>
<thead>
<tr>
<th>( E_{in} )</th>
<th>( V_V )</th>
<th>( r_V )</th>
<th>( a_V )</th>
<th>( W_V )</th>
<th>( r_WV )</th>
<th>( a_w )</th>
<th>( W_s )</th>
<th>( r_{WS} )</th>
<th>( a_s )</th>
<th>( V_{so} )</th>
<th>( r_{SO} )</th>
<th>( a_{SO} )</th>
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<td>5.2</td>
<td>1.22</td>
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<td>4.9</td>
<td>1.05</td>
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</tr>
<tr>
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<td>0.67</td>
<td>12.6</td>
<td>1.22</td>
<td>0.67</td>
<td>0.5</td>
<td>1.27</td>
<td>0.58</td>
<td>3.2</td>
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<tr>
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<td>0.1</td>
<td>1.27</td>
<td>0.58</td>
<td>2.2</td>
<td>1.05</td>
<td>0.60</td>
</tr>
</tbody>
</table>

### 2.2 Eikonal approximation

Eikonal approximation is very helpful in the study of proton-nucleus at high energies with respect to scattering the potential is much smaller than the energy since it reduce the differential equation to on variable, so it is used to solve and explain this problem. It is an quasi-classical approximation in the natural case is that every ray of the incident wave will change in phase when it passes through the potential of the path of a straight line, figure (1)[13].

\[
f(\theta) = \frac{k}{i} \int_0^\infty J_0(2kb \sin(\theta/2)) \left[ e^{i\chi(k,b)} - 1 \right] db
\]

where \( f(\theta) \) eikonal scattering amplitude, \( k \) relative momentum within the center of mass of the interaction between projectile and target. \( J_0 \) is the cylindrical Bessel function, \( b \) the impact parameter, \( \theta \) the scattering angle and \( \chi(k, b) \) is the phase shift function that can be found by interacting the optical potential in the z-direction as [16]:

\[
\chi(k,b) = -\frac{1}{2k} \int_0^\infty U(b)dz
\]
III. RESULTS AND DISCUSSION

Elastic scattering and inelastic scattering were performed and studied over a wide range of energies from 50 to 250 using optical potential. Calculation of the differential cross section depending on the eikonal approximation have been done based on the DWEIKO program[17]. This program displays several of the main results obtained after entering optimum optical potential parameters data based on the RIPL-3 data of the p+\(^{90}\)Zr reaction[18], which shows the parameters of the excited nucleus at different proton energy ≥ 50 MeV (50, 100, 156, 200 and 250 MeV), the value of root mean square (rms) of the nuclide is \(\langle r^2 \rangle^{1/2} = 4.349\) [fm]. The reason for using the intermediate and high energies is to gain nuclear structure information.

For total nuclear reaction cross-section the Glauber model tool up a confident technique to studied the structural properties of the p+\(^{90}\)Zr interaction[16]. Table (2) give the increasing for \(\sigma_T\) with energies, i.e. it was directly proportional to the formation probability of the nuclei. Anyway, we do not expect large difference in \(\sigma_T\) for high energies case but small value for 50 MeV energy case.

<table>
<thead>
<tr>
<th>(E_p) (MeV)</th>
<th>50</th>
<th>100</th>
<th>156</th>
<th>200</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_T) (mb)</td>
<td>462.02</td>
<td>1540.22</td>
<td>1706.56</td>
<td>1728.45</td>
<td>1722.86</td>
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</tbody>
</table>

The excitation cross-section directly linked with structure of nuclide information, so from the table (2) the cross-section was decrease as energy increase. Above 50 MeV or so, differences in the cross-sections become small as it expected because of the excitation energy of the first level is only represent 3.522 % from incident proton energy (50 MeV) and so on. Besides that, there are many partial waves combined with total spin \((J^s)\) that remove the difference in the target spin effect appears in the coupled channels where its independent in the calculations of optical potential

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>50</th>
<th>100</th>
<th>156</th>
<th>200</th>
<th>250</th>
</tr>
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<tbody>
<tr>
<td>(2p_{1/2})</td>
<td>0.699E+03</td>
<td>0.208E+03</td>
<td>0.110E+03</td>
<td>0.930E+02</td>
<td>0.109E+03</td>
</tr>
<tr>
<td>(1g_{9/2})</td>
<td>0.368E+01</td>
<td>0.238E+01</td>
<td>0.224E+01</td>
<td>0.223E+01</td>
<td>0.227E+01</td>
</tr>
<tr>
<td>Total</td>
<td>0.703E+03</td>
<td>0.211E+03</td>
<td>0.112E+03</td>
<td>0.952E+02</td>
<td>0.111E+03</td>
</tr>
</tbody>
</table>

**Figure 2:** Differential elastic cross-section comparison with result experimental calculated for different energies.

As the optical potential related with the nuclear density, then the elastic scattering is a good tool to test the nucleus structure specially at energies \(E \geq 50\) MeV for the proton (projectile). Calculations of proton+\(^{90}\)Zr elastic scattering using optical potentials were compared to the data with the experimental results of calculating the differential cross-section.
section and strength analysis at different energies [19]. Full measurements of the elastic scattering of 50 to 250 MeV protons from $^{90}$Zr are compared to data in figure (2). The step changes as the energy increases, forming the cross-section pass via resonances with energies that match the excited states of the nucleus of a compound. Since these states are a continuation of the bound states of the potential into the unbound region, their characteristics can be determined from the form of the potential [20][21]. Because of the transition of energy ($\Delta E$) from proton to target is small, as it the condition for satisfy eikonal approximation, on can note the behavior of the curves when the reaction start the scattering angle at once very small, then the wave function distorted.

![Figure 3: Real and imaginary optical potential parts as a function of radius at the nuclear surface for different energies of incoming proton.](image)

Optical potential calculations depend mainly on centripetal forces only, as they are given good overall for differential cross-sections, although the scattered nucleons are mainly polarized through the spin-orbit potential, which is not taken into account when making these calculations. It is clear from the figure (3) that the imaginary part has greater values than the real part at energy range. These results agree with [22]. The curves show that there is a strong increase at the surface of a nucleus due to an increase of the effective mass and the Pauli principle.

To describe the anisotropy of the angular distribution of gamma-ray one must introduced the anisotropy parameter $W(\theta)$. This parameter describe in accurately by three terms of Legendre polynomial ($P_0$, $P_2$, $P_4$) and given as:

$$W(\theta) = \frac{W_0}{4\pi} \left[ 1 + \alpha_2 P_2(\theta) + \alpha_4 P_4(\theta) \right]$$

(4)

where $\alpha$- is main gamma ray expansion coefficient found by least square method. For each incident proton there is an angle ($\theta_p$). For a discrete gamma ray of three distinct states and transitions were selected, the angular distribution of gamma, due to inelastic of proton interaction, was obtained for first excited state ($2^+$ at 2.3187 MeV) of $^{90}$Zr. In turn, there will be an angular distribution of gamma rays cross around the angle ($\theta_\gamma$) from 0° to 70° with data value smaller than 10° with respect to the incident angle for proton, as in figure (4). The results turn to the CM frame due to the correlation between gamma and particle, then weight of angular momentum distribution calculated which have a different values for each proton angle spanning the $\theta_p$, range from 20 to 70 [23].
A cross-section is a concept that plays a fundamental role in the study of atomic collision. The eikonal method was used in this work, which is valid at high incident energy. The cross-sections were measured using an optical model with partial-wave expansion that accurately reproduced the data up to 100 MeV. The measured differential cross sections display some minima above this energy value. The eikonal approximation reproduces the cross-section data well over all the angular range.

IV. CONCLUSION

Figure 4: Angular distribution of the gamma rays originating from the inelastic scattering of 50MeV proton energy by Zr nuclide in CM.

REFERENCE