TWO WAREHOUSE RED WINE INDUSTRY INVENTORY MODEL FOR DETERIORATING ITEMS WITH INFLATION AND WITHOUT SHORTAGES UNDER LIFO DISPATCHING POLICY

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ABSTRACT

A deterministic Red wine industry inventory model has been developed for deteriorating items having a ramp type demand with the effects of inflation with two-storage facilities under LIFO dispatching policy. The Red wine industry owned warehouse (OW) has a fixed capacity of W units; the Red wine industry rented warehouse (RW) has unlimited capacity under LIFO dispatching policy. Here, we assumed that the Red wine industry inventory holding cost in RW is higher than those in OW under LIFO dispatching policy. Shortages in Red wine industry inventory are allowed and partially backlogged and it is assumed that the Red wine industry inventory deteriorates over time at a variable deterioration rate under LIFO dispatching policy. The effect of inflation has also been considered for various costs associated with the Red wine industry inventory system. Numerical example is also used to study the behaviour of the model under LIFO dispatching policy. Cost minimization technique is used to get the expressions for total cost and other parameters under LIFO dispatching policy.

Keywords:- Inventory, owned warehouse, rented warehouse, ramp type demand, deteriorating items, inflation, without Shortages and LIFO dispatching policy.

I. INTRODUCTION

Many researchers extended the EOQ model to time-varying demand patterns. Some researchers discussed of Red wine industry inventory models with linear trend in demand. The main limitations in linear-time varying demand rate is that it implies a uniform change in the demand rate per unit time. This rarely happens in the case of any commodity in the market. In recent years, some models have been developed with a demand rate that changes exponentially with time. For seasonal products like clothes, Air conditions etc. at the end of their seasons the demand of these items is observed to be exponentially decreasing for some initial period. Afterwards, the demand for the products becomes steady rather than decreasing exponentially. It is believed that such type of demand is quite realistic. Such type situation can be represented by ramp type demand rate. An important issue in the Red wine industry inventory theory is related to how to deal with the unfulfilled demands which occur during shortages or stock outs. In most of the developed models researchers assumed that the shortages are either completely backlogged or completely lost. The first case, known as backordered or backlogging case, represent a situation where the unfulfilled demand is completely back ordered. In the second case, also known as lost sale case, we assume that the unfulfilled demand is completely lost. Furthermore, when the shortages occur, some customers are willing to wait for backorder and others would turn to buy from other sellers. In many cases customers are conditioned to a shipping delay and may be willing to wait for a short time in order to get their first choice. For instance, for fashionable commodities and high-tech products with short product life cycle, the willingness of a customer to wait for backlogging is diminishing with the length of the waiting time. Thus the length of the waiting time for the next replenishment would determine whether the backlogging would be accepted or not. In many real life situations, during a shortage period, the longer the waiting time is, the smaller is the backlogging rate would be. Therefore, for realistic business situations the backlogging rate should be variable.
and dependent on the waiting time for the next replenishment. Many researchers have modified Red wine industry inventory policies by considering the “time proportional partial backlogging rate”.

1. **ASSUMPTIONS AND NOTATIONS:**

In developing the mathematical model of the Red wine industry inventory system the following assumptions are being made:

1. A single item is considered over a prescribed period $T$ units of time.

2. The demand rate $D(t)$ at time $t$ is deterministic and taken as a ramp type function of time i.e.

$$
D(t) = \alpha_0 e^{-\alpha_2 (t-\tau_o)} H(t-\tau_o),
$$

where $\alpha_0 > 0$, $\alpha_2 > 0$, and $H(t-\tau_o)$ is the Heaviside’s function defined as

$$
H(t-\tau_o) = \begin{cases} 
0, & t < (\tau_o + t_L) \\
1, & t \geq (\tau_o + t_L)
\end{cases}
$$

3. The replenishment rate is infinite and lead-time is zero.

4. When the demand for goods is more than the supply. Shortages will occur. Customers encountering shortages will either wait for the vendor to reorder (backlogging cost involved) or go to other vendors (lost sales cost involved). In this model shortages are allowed and the backlogging rate is $\exp(-\alpha_3 t)$, when Red wine industry inventory is in shortage. The backlogging parameter $\alpha_3$ is a positive constant.

5. The variable rate of deterioration in both warehouse is taken as $\alpha_1(t) = \alpha_1 t$. Where $0 < \alpha_1 << 1$ and only applied to on hand Red wine industry inventory.

6. No replacement or repair of deteriorated items is made during a givencycle.

7. The Red wine industry owned warehouse (OW) has a fixed capacity of $W$ units; the Red wine industry rented warehouse (RW) has unlimited capacity.

8. The goods of OW are consumed only after consuming the goods kept in RW.

9. In addition, the following notations are used throughout this paper:

$$
\Pi_{ow}(t) \quad \text{The Red wine industry inventory level in OW at any time } t.
$$

$$
\Pi_{rw}(t) \quad \text{The Red wine industry inventory level in RW at any time } t.
$$

$$
\alpha_w \quad \text{The capacity of the own warehouse.}
$$

$$
Q \quad \text{The ordering quantity per cycle.}
$$

$$
T \quad \text{Planning horizon.}
$$

$$
\alpha_4 \quad \text{Inflation rate.}
$$

$$
\alpha_{hcow} \quad \text{The holding cost per unit per unit time in Red wine industry OW.}
$$

$$
\alpha_{hcrw} \quad \text{The holding cost per unit per unit time in Red wine industry RW.}
$$

$$
\alpha_{dc} \quad \text{The deterioration cost per unit.}
$$

$$
\alpha_{sc} \quad \text{The shortage cost per unit per unit time.}
$$
\( \alpha_{opc} \) The opportunity cost due to lost sales.

\( \alpha_{OC} \) The replenishment cost per order.

II. FORMULATION AND SOLUTION OF THE MODEL

The Red wine industry inventory levels at OW are governed by the following differential equations under LIFO dispatching policy:

\[
\frac{d\Pi_{ow}^{lifo}}{dt} = \left[ -\alpha_{1}(t)I_{lifo}^{lifo} \right] \quad 0 \leq t < (t_a + t_L)(1)
\]

\[
\frac{d\Pi_{ow}^{lifo}}{dt} + \alpha_{1}(t)I_{lifo}^{lifo} = -\alpha_{0}e^{-\alpha_{2}(t_a + t_L)}, \quad (t_a + t_L) \leq t \leq (t_1 + t_L)(2)
\]

And

\[
\left[ \frac{d\Pi_{ow}^{lifo}}{dt} \right] = \left[ -\alpha_{0}e^{-\alpha_{2}(t_a + t_L)}e^{-\alpha_{3}t} \right] \quad (t_1 + t_L) \leq t \leq (T + t_L)(3)
\]

with the boundary conditions,

\[
\Pi_{ow}^{lifo}(0) = \alpha_{w} \quad \text{and} \quad I_{lifo}^{lifo}(t_1 + t_L) = 0(4)
\]

The solutions of equations (1), (2) and (3) are given by

\[
\Pi_{ow}^{lifo}(t) = \alpha_{w}e^{-\alpha_{1}t^{2}/2}, \quad 0 \leq t < (t_a + t_L)(5)
\]

\[
\Pi_{ow}^{lifo}(t) = \left[ \alpha_{0}e^{-\alpha_{2}(t_a + t_L)} \frac{\left\{ (t_1 + t_L)^{-t} + \frac{1}{\alpha_{1}(t_1 + t_L)^{3} - \frac{3}{t^2}} \right\} e^{-\alpha_{1}t^{2}/2}}{6} \right] \quad (t_a + t_L) \leq t \leq (t_1 + t_L)(6)
\]

And

\[
\Pi_{ow}^{lifo}(t) = \left[ \frac{\alpha_{0}}{\alpha_{3}}e^{-\alpha_{2}(t_a + t_L)} \left\{ e^{-\alpha_{3}t} - e^{-\alpha_{3}(t_1 + t_L)} \right\} \right] \quad (t_1 + t_L) \leq t \leq (T + t_L)(7)
\]

respectively.

The Red wine industry inventory level at RW is governed by the following differential equations:

\[
\frac{d\Pi_{rw}^{lifo}}{dt} + \alpha_{1}(t)I_{lifo}^{lifo} = -\alpha_{0}e^{-\alpha_{2}t}, \quad 0 \leq t < (t_a + t_L)(8)
\]

With the boundary condition \( \Pi_{rw}^{lifo}(0) = 0 \) the solution of the equation (8) is
\[
\Pi_{\text{fio}}(t) = \begin{bmatrix}
\alpha_0 \\
\frac{\alpha_2}{2} \left( (t_\alpha + t_L)^2 - t^2 \right) + e^{-\alpha_1 t^2/2} \\
\frac{\alpha_1}{6} \left( (t_1 + t_L)^3 - t^3 \right)
\end{bmatrix} (t_\alpha + t_L) \leq t \leq (t_1 + t_L)
\] (9)

Due to continuity of \( \Pi_{\text{fio}}(t) \) at point \( t = (t_\alpha + t_L) \), it follows from equations (5) and (6), one has

\[
\alpha_w e^{-\alpha_1 (t_\alpha + t_L)^2/2} = \alpha_0 e^{-\alpha_2 (t_\alpha + t_L)} \left\{ \frac{(t_1 + t_L) - (t_\alpha + t_L)}{\alpha_1 (t_1 + t_L)^3 - (t_\alpha + t_L)^3} \right\} e^{-\alpha_1 (t_\alpha + t_L)^2/2}
\]

\[
\alpha_w = \alpha_0 e^{-\alpha_2 (t_\alpha + t_L)} \left\{ \frac{(t_1 + t_L) - (t_\alpha + t_L)}{\alpha_1 (t_1 + t_L)^3 - (t_\alpha + t_L)^3} \right\}
\] (10)

The total average cost consists of following elements:

(i) Ordering cost per cycle = \( \alpha_{OC} \) (11)

(ii) Holding cost per cycle (\( C_{HO} \)) in OW

\[
C_{HO} = \alpha_{hcow} \left\{ \int_0^{(t_\alpha + t_L)} \Pi_{\text{fio}}(t) e^{-\alpha_4 t} dt + \int_{(t_1 + t_L)}^{(t_\alpha + t_L)} \Pi_{\text{fio}}(t) e^{-\alpha_4 ((t_\alpha + t_L) + t)} dt \right\}
\]
\[ C_{HO} = \begin{cases} \alpha_{w} \left\{ \frac{(t_{a} + t_{L}) - \alpha_{4} (t_{a} + t_{L})^{2}}{2} - \frac{\alpha_{1} (t_{a} + t_{L})^{3}}{6} \right\} + \\
\frac{(t_{1} + t_{L})^{2}}{2} - \frac{\alpha_{4} (t_{1} + t_{L})^{3}}{6} + \\
\frac{\alpha_{4} (t_{1} + t_{L})^{4}}{12} - \frac{\alpha_{4} \alpha_{1} (t_{1} + t_{L})^{5}}{20} - \\
\frac{(t_{a} + t_{L})}{2} (2(t_{1} + t_{L}) - (t_{a} + t_{L})) - \\
\frac{\alpha_{4} (t_{a} + t_{L})}{24} (4(t_{1} + t_{L})^{3} - (t_{a} + t_{L})^{3}) + \\
\frac{\alpha_{4} \alpha_{1} (t_{a} + t_{L})^{2}}{6} (3(t_{1} + t_{L}) - 2(t_{a} + t_{L})) + \\
\frac{\alpha_{4} \alpha_{1} (t_{a} + t_{L})^{2}}{30} (5(t_{1} + t_{L})^{3} - 3(t_{a} + t_{L})^{3}) + \\
\frac{\alpha_{1} (t_{a} + t_{L})^{3}}{24} (4(t_{1} + t_{L}) - 3(t_{a} + t_{L})) \end{cases} \] (12)

(iii) Holding cost per cycle \((C_{HR})\) in RW

\[ C_{HR} = \left[ \alpha_{h_{crw}} \int_{0}^{t_{a} + t_{L}} \left\{ \frac{\alpha_{4} (t_{1} + t_{L})}{2} - \frac{\alpha_{1} (t_{a} + t_{L})^{3}}{6} \right\} e^{-\alpha_{4} t} dt \right] \]

\[ C_{HR} = \alpha_{h_{crw}} \alpha_{0} \left\{ \frac{(t_{a} + t_{L})^{2}}{2} - \frac{(3\alpha_{2} + \alpha_{4})}{6} (t_{a} + t_{L})^{3} + \right\} \] … (3.13)

(iv) Cost of deteriorated units per cycle \((C_{o})\)

\[ = \alpha_{dc} \left\{ \int_{0}^{t_{a} + t_{L}} \alpha_{1} t \Pi_{r_{w}}^{l_{10} (t)} e^{-\alpha_{4} t} dt + \right\} \]

\[ = \alpha_{dc} \left\{ \int_{0}^{t_{a} + t_{L}} \alpha_{1} t \Pi_{o_{w}}^{l_{10} (t)} e^{-\alpha_{4} t} dt + \right\} \int_{0}^{t_{1} + t_{L}} \alpha_{1} t \Pi_{o_{w}}^{l_{10} (t)} e^{-\alpha_{4} (t_{1} + t_{L})} dt \] (14)

(v) Shortage cost per cycle \((C_{s})\)
= \frac{1}{(T + t_L)} \left[ \text{Ordering cost} + \text{Holding cost in OW} + \text{Shortage cost} + \text{Opportunity cost} \right] = \frac{R(t + t_L)}{(T + t_L)} \tag{17}

\textbf{Appendix A(18)}

To minimize the total cost per unit time, the optimal values of \( t \) and \( T \) can be obtained by solving the following equations simultaneously

\[ \frac{\partial TC}{\partial t} = 0 \tag{19} \]

and

\[ \frac{\partial TC}{\partial (T + t_L)} = 0 \tag{20} \]

provided, they satisfy the following conditions

\[ \frac{\partial^2 TC}{\partial (t + t_L)^2} > 0, \quad \frac{\partial^2 TC}{\partial (T + t_L)^2} > 0 \]

and

\[ \left( \frac{\partial^2 TC}{\partial (t + t_L)^2} \right) \left( \frac{\partial^2 TC}{\partial (T + t_L)^2} \right) - \left( \frac{\partial^2 TC}{\partial (t + t_L) \partial (T + t_L)} \right)^2 > 0 \tag{21} \]
The equation (3.19) is equivalent to the following equation

\[
\frac{\partial TC}{\partial (t_1 + t_L)} = \frac{\alpha_0 - \alpha_2 (t_1 + t_L)}{(T + t_L)}
\]

Also equation (20) is equivalent to

\[
R = \frac{\alpha_0}{\alpha_3} \left( e^{-\alpha_4(T + t_L)} - \frac{\alpha_4}{\alpha_3} \right) + \frac{\alpha_2 (t_1 + t_L)}{\alpha_3 + \alpha_4} \left( e^{-\alpha_3(T + t_L)} - \frac{\alpha_4}{\alpha_3} \right) = 0 \quad (23)
\]

Equations (22) and (23) are highly nonlinear equations. Therefore, numerical solution of these equations can be obtained by using the software MATLAB 7.0.1.
III. PARTICULAR CASES: WITHOUT SHORTAGES:

When \( \alpha_3 \to \infty \) (i.e., the fraction of shortages backordered is zero), we get \( (T + t_L) \approx 0 \). The model reduces to the case where shortages are not allowed and hence the total average cost per unit time in equation (18) becomes:

\[
TC(t_1 + t_L) = \frac{1}{T + t_L} \left[ \text{Ordering cost} + \text{Holding cost} + \text{Deterioration cost} + \text{Shortage cost} + \text{Opportunity cost} \right] = \frac{R(t_1 + t_L)}{(T + t_L)}
\]

(24)

Appendix B(25)

The necessary condition to find the optimal solution of \( K(t_1) \) is

\[
\frac{\partial TC}{\partial (t_1 + t_L)} = \frac{\alpha_o}{(T + t_L)}
\]

\[
+ \alpha_{dc} \alpha_4 e^{-\alpha_2(t_1+L)} + \frac{e^{-\alpha_2(t_1+L) + \alpha_4}}{\alpha_4} \left( (t_1+L)^3 + \frac{\alpha_4(t_1+L)^2}{2} + \frac{\alpha_4(t_1+L)}{6} \right)
\]

\[
\frac{\partial^2 TC}{\partial (t_1 + t_L)^2} = \frac{\alpha_o}{(T + t_L)}
\]

\[
+ \alpha_{dc} \alpha_4 e^{-\alpha_2(t_1+L)} + \frac{e^{-\alpha_2(t_1+L) + \alpha_4}}{\alpha_4} \left( (t_1+L)^3 + \frac{\alpha_4(t_1+L)^2}{2} + \frac{\alpha_4(t_1+L)}{6} \right)
\]

(26)

Numerical Illustration:

To illustrate the model numerically the following parameter values are considered.

\[\alpha_1 = 0.0022 \text{ unit, } \alpha_0 = 502 \text{ units, } \alpha_2 = 0.22 \text{ unit, } \alpha_3 = 0.12 \text{ unit, } \alpha_4 = 0.052 \text{ unit, } \alpha_3 = 0.2 \text{ year}\]

\[\alpha_{OC} = Rs. 1002 \text{ per order, } \alpha_{hcow} = Rs. 0.02 \text{ per unit per year, } \alpha_{hcrw} = Rs. 2 \text{ per unit}\]

\[T = 12 \text{ year, } \alpha_{sc} = Rs. 2.0 \text{ per unit per year, } \alpha_{opc} = Rs. 42.0 \text{ per unit}\]

Then for the minimization of total average cost and with help of software the optimal policy can be obtained such as: \( t_1 = 0.599224 \text{ year, } S = 138.597235 \text{ units and } TC = Rs.758.115354 \text{ per year.}\)

IV. CONCLUSION

This study incorporates some realistic features that are likely to be associated with the Red wine industry inventory of any material under LIFO dispatching policy. Decay (deterioration) overtime for any material product and occurrence of shortages in Red wine industry inventory are natural phenomenon in real situations under LIFO dispatching policy. Red wine industry inventory shortages are allowed in the model. In many cases customers are conditioned to a shipping delay, and may be willing to wait for a short time in order to get their first choice. Generally speaking, the length of the waiting time for the next replenishment is the main factor for deciding whether the backlogging will be accepted or not. The willingness of a customer to wait for backlogging during a shortage period declines with the length of the waiting time. Thus, Red wine industry inventory shortages are allowed and partially backordered in the present chapter and the backlogging rate is considered as a decreasing function of the waiting time for the next replenishment under LIFO dispatching policy. Demand rate is taken as...
exponential ramp type function of time, in which demand decreases exponentially for the some initial period and becomes steady later on. Since most decision makers think that the inflation does not have significant influence on the Red wine industry inventory policy, the effects of inflation are not considered in some Red wine industry inventory models. However, from a financial point of view, an Red wine industry inventory represents a capital investment and must complete with other assets for a firm’s limited capital funds. Thus, it is necessary to consider the effects of inflation on the Red wine industry inventory system under LIFO dispatching policy. Therefore, this concept is also taken in this model. From the numerical illustration of the model, it is observed that the period in which Red wine industry inventory holds increases with the increment in backlogging and ramp parameters while Red wine industry inventory period decreases with the increment in deterioration and inflation parameters under LIFO dispatching policy. Initial Red wine industry inventory level decreases with the increment in deterioration, inflation and ramp parameters while Red wine industry inventory level increases with the increment in backlogging parameter. The total average cost of the system goes on increasing with the increment in the backlogging and deterioration parameters while it decreases with the increment in inflation and ramp parameters under LIFO dispatching policy. The proposed model can be further extended in several ways under LIFO dispatching policy. For example, we could extend this deterministic model in to stochastic model. Also, we could extend the model to incorporate some more realistic features, such as quantity discount or the unit purchase cost, the Red wine industry inventory holding cost and others can also taken fluctuating with time under LIFO dispatching policy.

REFERENCES


Appendix A

\[
\begin{align*}
&= a_{d_0} a_1 \\
&= a_0 \left( \frac{1}{6} (t_\alpha + t_L)^3 - \left( \frac{\alpha_2}{4} + \frac{\alpha_4}{12} \right) (t_\alpha + t_L)^4 + \left( \frac{\alpha_1}{40} + \frac{\alpha_4 a_2}{15} \right) (t_\alpha + t_L)^5 - \left( \frac{\alpha_4 a_1}{36} - \frac{\alpha_2 a_1}{24} \right) (t_\alpha + t_L)^6 \right) \\
&+ a_w \left( \frac{(t_\alpha + t_L)^2}{2} - \frac{\alpha_1}{3} (t_\alpha + t_L)^3 - \frac{\alpha_4}{8} (t_\alpha + t_L)^4 \right)
\end{align*}
\]

\[
\begin{align*}
&= \frac{(t_1 + t_L)^3}{6} - \frac{\alpha_4}{12} (t_1 + t_L)^4 + \frac{a_1}{40} (t_1 + t_L)^5 - \frac{\alpha_4 a_1}{36} (t_1 + t_L)^6 - \frac{\alpha_1}{6} (t_\alpha + t_L)^2 (3(t_1 + t_L) - 2(t_\alpha + t_L)) - \frac{\alpha_4}{12} (t_\alpha + t_L)^3 (4(t_1 + t_L) - 3(t_\alpha + t_L)) - \frac{\alpha_4 a_1}{36} (t_\alpha + t_L)^3 (2(t_1 + t_L)^3 - (t_\alpha + t_L)^3) - \frac{\alpha_1}{40} (t_\alpha + t_L)^4 (5(t_1 + t_L) - 4(t_\alpha + t_L))
\end{align*}
\]
Appendix B

\[ K(t_1 + t_L) = \frac{1}{(T + t_L)} a_{\infty} + a_{\text{cow}} \]

\[ + a_{\text{hcr}} a_{t_1} \left( \frac{t_1 + t_L}{2} \right) \left( 3a_2 + a_4 \right) \left( t_1 + t_L \right)^3 + \frac{a_1 a_4}{12} a_2 \left( t_1 + t_L \right)^4 - \frac{a_1 a_2}{20} \left( t_1 + t_L \right)^5 \]

\[ + a_{d_2} a_{t_1} \left( \frac{1}{6} \left( t_1 + t_L \right)^3 \right) \left( \frac{a_2}{4} + \frac{a_4}{12} \right) \left( t_1 + t_L \right)^4 + \frac{a_1 a_4}{40} \left( t_1 + t_L \right)^5 - \frac{a_1 a_2}{36} \left( t_1 + t_L \right)^6 \]

\[ + a_{d_3} a_{t_1} \left( \frac{t_1 + t_L}{2} \right) \left( \frac{a_2}{3} a_4 \right) \left( t_1 + t_L \right)^3 + \frac{a_1 a_4}{8} \left( t_1 + t_L \right)^4 \]

\[ \alpha_1 e^{-a_2 (t_1 + t_L)} a_{a_2} \left( \frac{t_1 + t_L}{2} \right)^2 - \alpha_1 \left( t_1 + t_L \right)^3 \]

\[ + \frac{a_4}{6} \left( t_1 + t_L \right)^2 - \frac{a_4}{6} \left( t_1 + t_L \right)^3 + \frac{a_1}{12} \left( t_1 + t_L \right)^3 + \frac{a_1}{12} \left( t_1 + t_L \right)^5 \]

\[ - \frac{a_4 a_1}{20} \left( t_1 + t_L \right)^5 - \frac{a_4 a_1}{30} \left( t_1 + t_L \right)^5 \]

\[ - \frac{a_4 a_1}{40} \left( t_1 + t_L \right)^5 - \frac{a_4 a_1}{36} \left( t_1 + t_L \right)^6 \]

\[ + \frac{a_4 a_1}{8} \left( t_1 + t_L \right)^5 \]

\[ - \frac{a_4 a_1}{12} \left( t_1 + t_L \right)^3 \]