Application Of Sagemath, Python Coding In Vertex – Edge And Efficient Vertex - Edge Dominating Sets In Graphs, Fuzzy Graphs

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Abstract:
Graph theory is one of the most flourishing branches of mathematics with applications to wide variety of subjects which is used to model many types of relations among any physical, real time situation. In many real world problem we get only partial information about the problem, the vagueness in the description and uncertainty has led to the growth of fuzzy graph theory. A mathematical frame work to describe uncertainty in real life situation was first suggested by L. A. Zadeh. In 1975 Rosenfeld introduced fuzzy graph theory. After the pioneering work of Rosenfeld several authors has been finding deeper results, and fuzzy analogs of several graph theoretic concepts. One such interesting graph theoretic concept is domination in graphs. The majority of research on the domination parameter deals with the set of vertices that dominate other vertices or sets of edges that dominate other edges. There has been little research on vertices which dominates the edges which are incident and adjacent to incident edges, also efficiency of the domination parameter is implemented. This Study proposed the concept of determining the efficient vertex edge domination number of $G$ and $\bar{G}$ and extended the concept to fuzzy graph. The exact values of Total efficient vertex edge domination number of several family of fuzzy graphs and special family of graphs, Total graph, are determined, and compared the efficient vertex edge domination number of family of graphs, concluded that the (cardinality) domination number remains same. SageMath software is used to construct Total graph and Python coding is generated to find the Vertex edge dominating vertices if the graph structure is given.

Key words:  Total graph, n – Barbell graph, Fuzzy graphs, Efficient vertex edge Domination, SageMath.

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AMS Classification: Paths and cycles 05C38, Connectivity of graph 05C40, Dominating set of graph 05C69, Graph Operations 05C76, Graph Representations 05C62.

1. Introduction:
Let \( G = (V, E) \) be a graph with a vertex set \( V(G) \) and edge set \( E(G) \). A dominating set \( D \) is said to be efficient domination set if for every vertex \( u \in V, |N[u] \cap S| = 1 \). Equantly a dominating set \( D \) is efficient if the distance between any two vertices in \( S \) is at least 4, \( d(u,v) \leq 4 \). The minimum cardinality of a minimal efficient dominating set is called efficient domination number of a graph \( G \) and is denoted by \( \gamma_e(G) \).

1.1 Vertex – edge domination:
A set \( S \subseteq V(G) \) is a vertex-edge dominating set if for all edges \( e \in E(G) \), there exist a vertex \( v \in S \) such that \( v \) ve-dominates \( e \). Otherwise for a graph \( G = (V,E) \) a vertex \( u \in V(G) \) ve-dominates an edge \( vw \in E(G) \) if
(i) \( u = v \) or \( u = w \) (\( u \) is incident to \( vw \)), or
(ii) \( uv \) or \( uw \) is an edge in \( G \) (\( u \) is incident to an edge that is adjacent to \( vw \)).
A subset \( S \) of \( V \) is said to be minimal vertex-edge dominating set if no proper subset of \( S \) is a vertex-edge dominating set of \( G \). The minimum cardinality of a minimal vertex-edge dominating set is called vertex-edge domination number of a graph \( G \) and is denoted by \( \gamma_{ve}(G) \).

Example:

1.2 Efficient Vertex – edge domination:
A subset \( S \) of \( V \) is efficient vertex-edge dominating set if \( S \) is vertex-edge dominating set such that if for all edges \( e \in E(G) \), there exist a vertex \( v \in S \) such that \( v \) ve-dominates \( e \) for every vertex \( u \in V, |N[u] \cap S| = 1 \). The minimum cardinality of an efficient vertex-edge dominating set is called Efficient Vertex-edge domination number of a graph \( G \) and is denoted by \( \gamma_{eve}(G) \).

Example:
Crisp Graph

**Python coding:**
Python coding has been developed to construct the above graph also to locate the vertex edge dominating vertices of the same graph, by giving an input of Number of vertices and the edge connections.
1.2 Fuzzy graph:
- Let $V$ be a finite non empty set, let $E$ be the collection of all two element subsets of $V$. A fuzzy graph $G = (\sigma, \mu)$ is a set with 2 functions $\sigma$: $V \rightarrow [0, 1]$ and $\mu$: $V \times V \rightarrow [0, 1]$, such that $\mu(x, y) \leq \sigma(x) \land \sigma(y)$ for all $x, y \in V$.
- The underlying crisp graph of a fuzzy graph $G = (\sigma, \mu)$ is denoted by $G^* = (\sigma^*, \mu^*)$ where $\sigma^* = \{ v_i \in V/ \sigma(v_i) > 0 \}$ and $\mu^* = \{ (v_i, v_j) \in V \times V/ \mu(v_i, v_j) > 0 \}$.
- The order $p$ and size $q$ of a fuzzy graph $G = (\sigma, \mu)$ is defined as $p = \sum \sigma(x)$ and $q = \sum \mu(xy)$.
- The degree of vertex $u$ is defined as the sum of the weights of the edges incident at $u$ and is denoted by $d(u)$.

1.3 Observations:
In this full paper we have named each vertex $\{v_1, v_2, v_3, \ldots, v_{10}, v_{11}\}$ of crisp graph $G$ as $\{0.1, 0.2, 0.3, \ldots, 1, 0.1\}$ in fuzzy graph, similarly we have named the edges too.
- The dominating set of above graph is $D = \{ v_3(0.3), v_6(0.6), v_9(0.9) \}$ and domination number is $\gamma(G) = 1.8$.
- The vertex edge dominating set is $S = \{ v_3(0.3), v_9(0.9) \}$ and vertex edge domination number is $\gamma_{ve}(G) = 1.1$.
- The efficient vertex edge dominating set is $S = \{ v_3(0.3), v_9(0.9) \}$ and the efficient vertex edge.
The domination number is $\gamma_{eve}(G) = 1.1$

- From the above graph we observe that
  
  i) $\gamma(G) > \gamma_{ve}(G)$,
  
  ii) $\gamma(G) > \gamma_{eve}(G)$ and
  
  iii) $\gamma_{ve}(G) = \gamma_{eve}(G)$

- **Total graph**: The total graph $T(G)$ of a graph $G$ is the graph whose vertices correspond to the vertices and edges of $G$, and whose two vertices are joint if and only if the corresponding vertices are adjacent, edges are adjacent or vertices and edges are incident in $G$.

**Example:**

![Graphs]

**SageMath software** is a free open-source mathematics software. It builds on top of many existing open-source packages, and access through a common, **Python-based language**.

To construct the total graph of Path $P_4$ and cycle $C_4$, we are using SageMath, the following are the code gives the Total graph structure.

**$T(P_4)$ – SageMath Code and Graph Structure:**

```python
p3 = graphs.PathGraph(4)
p3.add_vertices([4, 5, 6])
p3.vertices()
for i in range(2):
    p3.add_edges([(i+4, i), (i+4, i+1), (i+4, i+5)])
p3.add_edges([(6, 2), (6, 3)])
p3.show()
[0, 1, 2, 3, 4, 5, 6]
```
**T(C₄) – SageMath Code and Graph Structure:**

```python
p4=graphs.CycleGraph(4)
show(p4)
p4.dominating_set()
for i in range(3):
    p4.add_edges([(i+4,i),(i+4,i+1),(i+4,i+5)])
p4.add_edges([(7,0),(7,3),(7,4)])
p4.show()
p4.dominating_set()```

**Python coding:** Python coding has been developed to construct the Total graph of cycle C₉ also to locate the vertex edge dominating vertices by giving an input of Number of vertices and the edge connections in notepad.
**n – Barbell Graph:** The n – Barbell graph is a special type of undirected graph consisting of two non-overlapping n-vertex cliques together with a single edge that has an end point in each clique. n – barbell graph is the simple graph obtained by connecting two copies of a complete graph $K_n$ by a bridge.

**Example:**
Observation:

- Vertex – Edge domination number of n – barbell graph is 2
- Efficient Vertex – Edge domination number of n – barbell graph does not exist as the efficiency condition does not satisfied.

2. Efficient Vertex – Edge dominating set of graph:
In this chapter we will discuss about efficient vertex edge domination number of some family of graph such as path, cycle, complete graph, Total graph, and Jahangir graph by converting the Crisp Graph into Fuzzy graph.

Theorem: 2.1 For any graph G the upper and lower bound is $1 \leq \gamma_{eve}(G) \leq \frac{n}{4}$

Proof:
Let G be a graph with vertex set $V(G)$ and edge set $E(G)$.
To prove the lower bound, Let us consider a simple connected graph with 3 vertices and 2 edges, one vertex will dominate both the incident edge and adjacent to incident edge. Therefore Lower bound is proved. That is $1 \leq \gamma_{eve}(G)$.
To prove the upper bound, Let G be a simple connected graph with n vertices, since Every edge is incident with exactly two vertices, and each edge as two adjacent edges. one vertex will dominate both the incident edges and adjacent to incident edge. This implies one vertex will cover 4 edges of G, that is $\frac{n}{4}$ number of vertices s are needed to cover all the $2n-1$ edges of a graph with n vertices. Therefore $\gamma_{eve}(G) \leq \frac{n}{4}$. Thus $1 \leq \gamma_{eve}(G) \leq \frac{n}{4}$ for all n ≥ 3.

Result: 2.2 The efficient vertex edge domination number of Path is
(i) $\gamma_{eve}(P_{4n+4}) = n + 1$ for all $n = 1, 2, 3, \ldots$
(ii) $\gamma_{eve}(P_{4n+2}) = \left\lfloor \frac{4n+2}{3} \right\rfloor$ for all $n = 1, 2, 3, \ldots$
(iii) $\gamma_{eve}(P_{4n+3}) = \left\lfloor \frac{4n+3}{3} \right\rfloor$ for all $n = 1, 2, 3, \ldots$

SageMath Code to find the efficient ve - domination number of Path $P_n$: 

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Observation: \( \gamma \) in Fuzzy Path

# Efficient vertex edge domination number#

**Case : 1**

\[ f(n) = \frac{4n+2}{3} \]
\[ F(n) = 4n + 2 \]

```python
print("The vertex edge domination number of Path with")
F(23)
print(" vertices is :")
floor(f(23))
```

The vertex edge domination number of Path with 94 vertices is: 31

**Case : 2**

\[ f(n) = \frac{4n+3}{3} \]
\[ F(n) = 4n + 3 \]

```python
print("The vertex edge domination number of Path with")
F(4)
print(" vertices is :")
floor(f(4))
```

The vertex edge domination number of Path with 19 vertices is: 6

**Case : 3**

\[ f(n) = n + 1 \]
\[ F(n) = 4n + 4 \]

```python
print("The vertex edge domination number of Path with")
F(3)
print(" vertices is :")
floor(f(3))
```

The vertex edge domination number of Path with 16 vertices is: 4

**Case : 4**

\[ f(n) = n \]
\[ F(n) = 3n \]

```python
print("The vertex edge domination number of Path with")
F(5)
print(" vertices is :")
floor(f(5))
```

The vertex edge domination number of Path with 15 vertices is: 5

**Observation:** \( \gamma \) in Fuzzy Path
The efficient vertex edge domination number of Fuzzy Path:

(i) \( \gamma_{eve} (P_3) = \gamma_{eve} (P_4) = \gamma_{eve} (P_5) = 0.3 \)

(ii) \( \gamma_{eve} (P_7) = \gamma_{eve} (P_8) = \gamma_{eve} (P_9) = 0.3 + 0.7 = 1 \)

(iii) \( \gamma_{eve} (P_{11}) = \gamma_{eve} (P_{12}) = \gamma_{eve} (P_{13}) = 0.3 + 0.7 + 0.1 = 1.1 \)

**Result:** 2.3 The efficient vertex edge domination number of Cycle is \( \gamma_{eve} (C_{4n}) = n \) for all \( n \)

**Observation:** \( \gamma_{eve} \) in Fuzzy Cycle

- The efficient vertex edge domination number of Fuzzy Cycle is
  \( \gamma_{eve} (C_4) = 0.1 \);
  \( \gamma_{eve} (C_8) = 0.1 + 0.5 = 0.6 \)
  \( \gamma_{eve} (C_{12}) = 0.1 + 0.5 + 0.9 = 1.5 \); and \( \gamma_{eve} (C_{16}) = 2.5 \)

**Result:** 2.5 The efficient vertex edge domination number of Complete graph is \( \gamma_{eve} (K_n) = 1 \).

**Observation:** \( \gamma_{eve} \) in Fuzzy Cycle

- The efficient vertex edge domination number of Fuzzy Complete graph is
  \( \gamma_{eve} (K_n) = 0.1 \)

**Result:** 2.6 The efficient vertex edge domination number of Complete bi partite graph is
\( \gamma_{eve} (K_{n,m}) = 1 \).

**Observation:** \( \gamma_{eve} \) in Fuzzy Cycle

- The efficient vertex edge domination number of Fuzzy Complete bi partite graph is
  \( \gamma_{eve} (K_{n,m}) = 0.1 \).

**Result:** 2.7 The efficient vertex edge domination number of Star graph is \( \gamma_{eve} (K_{n-1,1}) = 1 \).

**Observation:** \( \gamma_{eve} \) in Fuzzy Cycle

- The efficient vertex edge domination number of Fuzzy Star graph,
  \( \gamma_{eve} (K_{n-1,1}) = 0.1 \).

**Conclusion:**

The concept of efficient vertex edge domination parameter was introduced and identified the eve domination number for some well-known family of graphs. It is interesting to find the bounds for efficient vertex edge domination. Extended our work in Fuzzy Graphs, since many real world problem we get only partial information about the problem, the vagueness in the description and uncertainty has led to the growth of fuzzy graph theory. To construct the total graph of Path and cycle a trial has been made using SageMath software. Python code has been developed to find vertex edge domination number and vertices.

As a future work, aiming in extending our work to find the eve domination number of special graphs Total graph and Jahangir graph, and working in programming language to setup codes in Python to get the eve domination number of any graphs directly.

**References:**

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