ANALYSIS OF INVENTORY MANAGEMENT FOR DETERIORATING ITEMS UNDER FIFO AND LIFO DISPATCHING POLICY (A COMPARATIVE STUDY OF TWO WAREHOUSES)

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ABSTRACT:
In the majority of the literatures on two-warehouses inventory choice models, the last in-first-out (LIFO) dispatching strategy has consistently been accepted. This assumption, be that as it may, isn't in line with actual practise of most business substances. To improve the newness of product or merchandise, organizations ordinarily follow the first-in-first-out technique (FIFO). This irregularity frames the base and main inspiration for our exploration. Correlation of the two models showed that the FIFO model is more affordable to work than LIFO, if the blended impacts of deterioration & holding cost in RW are not as much as that of OW. At last, numerical example are given to research and look at the effect that different parameters have on strategy decision.

Keywords: LIFO, FIFO, inventory.

I. INTRODUCTION
The impact of deterioration is significant in many inventory frameworks. A large portion of the physical products go through rot or crumbling after some time. Disintegration of products in stock is a practical component &inventory modelers wanted to null over this factor. Zeynep [1] Stock administration under replaceable interest: A stochastic framework situation issue of replaceable ware stock is examined utilizing stochastic game hypothesis. Balkhi [2, 3] audited stock models with ceaseless interest at regular decay rates just as recharging remittances. Zhou et al. [4] & Kumar et al [5] built up a two-store model for item disintegrating by tolerating intrigue feature, which isn't simply time-subordinate, yet moreover the stock level under vastness and sensible lack. Tavaet al.[6] author must test circumstances where a person who experiences a bookkeeping stock at an organization is probably going to foliate the matter of the firm. Pakkala et al. [7, 8] planned a model for obscuring things with monstrosity release technique where two indisputable appropriation habitats were used. Hsieh et al [9] suggested two dispersion habitats thought of best mentioning approach by raising net current assessment of full scale utilization.

II. ASSUMPTION
i. Replenishment rate is quick.
ii. Lead-time is unimportant.
iii. The arranging horizon of the stock framework is infinite.

III. NOTATION
A: replenishment cost for each request c: buying cost per unit
QF, QL: request amount per cycle for FIFO & LIFO separately
SF, SL: maximum stock level per cycle for FIFO & LIFO separately

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IV. MODEL DESCRIPTION AND ANALYSIS

Case 1: When $t_d < t_w$

During the time stretch $[0, t_d]$, there is no disintegration. Along these lines, the inventory in OW I o (t) is exhausted uniquely because of interest while in RW, inventory level continues as before. Further, during the time interval $[t_d, t_w]$ the inventory level in OW is dropping to zero because of the joined impact of interest and decay and the stock in RW Ir (t) gets drained because of weakening as it were. Presently, during the time stretch $[t_w, t_r]$ consumption of inventory Ir (t) happens in RW because of consolidated impact of interest and crumbling and it scopes to zero at time. Besides, during the span $[t_r, T]$ the interest is multiplied. In this way, B (t) speaks to the degree of negative stock at time t in the interval $[t_r, T]$. The behaviour of the model in the time interval $[0, T]$ has been shown to graphically in figure 1.

![Figure 1: Two-warehouse FIFO inventory system, when $t_d > t_w$](image)

Therefore, the differential conditions that describe the level of stock in RW & OW at time t for period $(0, T)$ are set as follows:

\[
\frac{dI_0(t)}{dt} = -D; \quad \text{for } 0 \leq t \leq t_d \tag{1}
\]

\[
\frac{dI_0(t)}{dt} + aI_0(t) = -D; \quad \text{for } t_d \leq t \leq t_w \tag{2}
\]

\[
\frac{dI_r(t)}{dt} + \beta I_r(t) = 0; \quad \text{for } t_d \leq t \leq t_w \tag{3}
\]

\[
\frac{dI_r(t)}{dt} + \beta I_r(t) = -D; \quad \text{for } t_w \leq t \leq t_r \tag{4}
\]

\[
\frac{dB(t)}{dt} = De^{-\delta(T-t)}; \quad \text{for } t_r \leq t \leq T \tag{5}
\]

Solutions of above five differential conditions (1), (2), (3), (4) and (5) with boundary conditions res. $I_0(0) = W$, $I_0(t_w) = W$, $I_r(t_d) = S_f - W$, $I_r(t_r) = 0$ & $B(t_r) = 0$

\[
I_0(t) = W - Dt; \quad \text{for } 0 \leq t \leq t_d \tag{6}
\]

\[
I_0(t) = \frac{D}{a} \left(e^{\alpha(t_w-t)} - 1\right); \quad \text{for } t_d \leq t \leq t_w \tag{7}
\]

\[
I_r(t) = (S_f - W)e^{\beta(t_d-t)}; \quad \text{for } t_d \leq t \leq t_w \tag{8}
\]

\[
I_r(t) = \frac{D}{\beta} \left(e^{\beta(t_r-t)} - 1\right); \quad \text{for } t_w \leq t \leq t_r \tag{9}
\]

\[
B(t) = \frac{D}{\delta} \left(e^{-\delta(T-t)} - e^{-\delta(T-t_r)}\right); \quad \text{for } t_r \leq t \leq T \tag{10}
\]

Number of lost sales at time t

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$$L(t) = D \left[ (t - t_r) - \frac{1}{\delta} \left( e^{-\delta(t-t_r)} - e^{-\delta(t-t_r)} \right) \right]$$  \hspace{1cm} (11)$$

Considering coherence of $I_0(t)$ at $t = t_d$, it follows from conditions (6) & (7)

$$t_w = t_d + \frac{1}{\alpha} \ln \left| 1 + \frac{d}{D}(W - D t_d) \right|$$  \hspace{1cm} (12)$$

Considering coherence of $I_r(t)$ at $t = t_w$, it follows from conditions (8) & (9)

$$S_r = W + \frac{D}{\beta} \left( e^{\beta(t_r-t_d)} - e^{\beta(t_w-t_d)} \right)$$  \hspace{1cm} (13)$$

Putting $t = T$ in above eq. (10),

$$B(T) = \frac{D}{\delta} \left( 1 - e^{\delta(T-t_r)} \right)$$  \hspace{1cm} (14)$$

Using equ. (12) & (14)

$$Q_f(t) = W + \frac{D}{\beta} \left( e^{\beta(t_r-t_d)} - e^{\beta(t_w-t_d)} \right) + \frac{D}{\delta} \left( 1 - e^{\delta(T-t_r)} \right)$$ \hspace{1cm} (15)$$

Hence, various costs during cycle ($0, T$) are evaluated as follows:

a) Ordering cost per cycle = $A$

b) The inventory holding cost per cycle in RW

$$HC_{rw} = F \left[ \int_0^{t_d} I_r(t)dt + \int_{t_d}^{t_w} I_r(t)dt + \int_{t_w}^{t_r} I_r(t)dt \right]$$

$$= \frac{FD}{\beta} \left[ e^{\beta(t_r-t_d)} - e^{\beta(t_w-t_d)} \right] \left\{ t_d + \frac{1}{\beta} \left( 1 - e^{\delta(t_d-t_w)} \right) \right\} + \left\{ \frac{1}{\beta} e^{\beta(t_r-t_w)} - 1 \right\} - (t_r - t_w) \right\}$$

c) The inventory holding cost per cycle in OW

$$HC_{ow} = H \left[ W t_d - \frac{D t_d^2}{2} - \frac{D}{\alpha} \left( 1 - e^{\alpha(t_w-t_d)} \right) + (t_w - t_d) \right]$$

d) The backlogged cost per cycle is

$$SC = s \frac{D}{\delta} \left[ \frac{1}{\delta} + T - t_r \right] e^{-\delta(T-t_r)}$$

e) The opportunity cost due to lost sale is

$$= c_1 D \left[ T - t_r - \frac{1}{\delta} \left( 1 - e^{-\delta(T-t_r)} \right) \right]$$

f) The deterioration cost per cycle

$$= c \left[ \int_{t_d}^{t_r} I_r(t)dt + \int_{t_d}^{t_w} I_0(t)dt \right]$$
Now, total relevant cost per unit time during \( (0, T) \) using conditions is given by

\[
TC_{f1}(t_r, T) = \frac{1}{T} \left[ A + \frac{PD}{\beta} \left( e^{\beta(t_r-t_d)} - e^{\beta(t_w-t_d)} \right) \left( t_d + \frac{1}{\beta} \left( 1 - e^{\beta(t_d-t_w)} \right) \right) + \frac{1}{\beta} \left( e^{\beta(t_r-t_w)} - 1 \right) - (t_r - t_w) \right] + H \left[ Wt_d - \frac{D^2}{2} - \frac{D}{\alpha} \left( 1 - e^{\alpha(t_w-t_d)} \right) + (t_w - t_d) \right] + \frac{D}{\delta} \left( 1 - e^{-\delta(T-t_r)} \right) + c_D D \left( T - t_r - \frac{1}{\delta} \left( 1 - e^{-\delta(T-t_r)} \right) \right) + cD \left( \frac{1}{\alpha} \left( 1 - e^{\alpha(t_w-t_d)} \right) + (t_w - t_d) + \frac{1}{\beta} \left( e^{\beta(t_r-t_d)} - e^{\beta(t_w-t_d)} \right) \left( 1 - e^{\beta(t_d-t_w)} \right) + \frac{1}{\beta} \left( e^{\beta(t_r-t_w)} - 1 \right) - (t_r - t_w) \right] \right]
\]

(16)

**Case 2: When** \( t_d > t_w \)

For this situation, time during which no weakening happens is more prominent than the time during which stock in OW gets zero and the conduct of the model over the time span \( [0, T] \) has been graphically spoken to beneath in Figure 2.

![Figure 2: Two-warehouse FIFO inventory system when \( t_d > t_w \)](image)

Subsequently, the differential conditions that portray the stock level in the RW and OW at time \( t \) for period \( (0, T) \) are determined by the formula:

\[
\frac{dI_0(t)}{dt} = -D; \quad \text{for } 0 \leq t \leq t_w
\]

(17)

\[
\frac{dI_r(t)}{dt} + \beta I_r(t) = 0; \quad \text{for } t_w \leq t \leq t_d
\]

(18)

\[
\frac{dI_r(t)}{dt} + \beta I_r(t) = -D; \quad \text{for } t_d \leq t \leq t_r
\]

(19)

\[
\frac{dB(t)}{dt} = De^{-\delta(T-t)}; \quad \text{for } t_r \leq t \leq T
\]

(20)

Solutions of above five differential conditions (17), (18), (19), (20) with \( I_0(0) = W I_0(t_w) = W I_r(t_d) = S_f - W \), \( I_r(t_r) = 0 \) and \( B(t_r) = 0 \), Res.

\[
I_0(t) = W - Dt \quad \text{for } 0 \leq t \leq t_w
\]

(21)

\[
I_r(t) = (S_f - W) e^{\beta(t_d-t)}; \quad \text{for } t_w \leq t \leq t_d
\]

(22)

\[
I_r(t) = \frac{D}{\beta} \left( e^{\beta(t_r-t)} - 1 \right); \quad \text{for } t_d \leq t \leq t_r
\]

(23)

\[
B(t) = \frac{D}{\delta} \left( e^{-\delta(T-t)} - e^{-\delta(T-t_r)} \right); \quad \text{for } t_r \leq t \leq T
\]

(24)
Number of lost sales at time \( t \) is

\[
L(t) = D \left[ (t - t_r) - \frac{1}{\delta} \left( e^{\delta(T-t)} - e^{-\delta(T-t_r)} \right) \right]
\]  
(25)

Considering continuity of \( I_r(t) \) at \( r = t_d \) it follows from equ. (22) & (23) that

\[
S_r = W + \frac{D}{\beta} \left( e^{\beta(t_r-t_d)} - 1 \right) + D(t_d - t_w)
\]  
(26)

Putting \( t = T \) in equation (24),

\[
B(T) = \frac{D}{\delta} \left( 1 - e^{-\delta(T-t_r)} \right)
\]  
(27)

Using ;(26) and (27)

\[
Q_f = W \frac{D}{\beta} \left( e^{\beta(t_r-t_d)} - 1 \right) + D(t_d - t_w) + \frac{D}{\delta} \left( 1 - e^{-\delta(T-t_r)} \right)
\]  
(28)

Total cost per cycle consists of following elements:

a) Ordering cost per cycle = \( A \)

b) The inventory holding cost per cycle in RW

\[
HC_{rw} = FD \left[ \frac{1}{\beta} \left( e^{\beta(t_r-t_d)} - 1 \right) + (t_d - t_w) \right] t_d + \frac{1}{\beta^2} \left( e^{\beta(t_r-t_d)} - 1 \right) - \frac{(t_d - t_w)^2}{2} - \frac{(t_r - t_d)}{\beta}
\]

c) Inventory holding cost per cycle in OW

\[
HC_{ow} = H \int_0^{t_w} l_0(t) dt = \frac{HDt_w^2}{2}
\]

d) The backlogged cost per cycle is

\[
SC = s \frac{D}{\delta} \left[ \frac{1}{\delta} - \frac{1}{\delta} \left( 1 + T - t_r \right) e^{-\delta(T-t_r)} \right]
\]

e) The opportunity cost due to lost sale is

\[
= c_1 D \left[ T - t_r - \frac{1}{\delta} \left( 1 - e^{-\delta(T-t_r)} \right) \right]
\]

f) Deterioration cost per cycle

\[
c_\beta \int_{t_d}^{t_r} I_r(t) dt = cD \frac{1}{\beta} \left( e^{\beta(t_r-t_d)} - 1 \right) - (t_r - t_d)
\]

Now, total relevant cost per unit time during cycle \((0, T)\)

\[
TC_{f2}(t_r, T) = \frac{1}{T} \left[ A + \frac{HDt_w^2}{2} + FD \left\{ \frac{1}{\beta} \left( e^{\beta(t_r-t_d)} - 1 \right) + (t_d - t_w) \right\} t_d + \frac{1}{\beta^2} \left( e^{\beta(t_r-t_d)} - 1 \right) - \frac{(t_d - t_w)^2}{2} - \frac{(t_r-t_d)}{\beta} \right] + \frac{sd}{\delta} \left[ \frac{1}{\delta} - \frac{1}{\delta} \left( 1 + T - t_r \right) e^{-\delta(T-t_r)} \right] + c_1 D \left[ T - t_r - \frac{1}{\delta} \left( 1 - e^{-\delta(T-t_r)} \right) \right] + cD \left\{ \frac{1}{\beta} \left( e^{-\delta(t_r-t_d)} - 1 \right) - \frac{(t_r-t_d)}{\beta} \right\}
\]  
(29)
Therefore, total relevant cost per unit time during cycle (0, T)

\[ TC_f(t_r, T) = \begin{cases} TC_{f1}(t_r, T) & \text{if } t_d \leq t_w \\ TC_{f2}(t_r, T) & \text{if } t_d \leq t_w \end{cases} \]  

(30)

\[ t_w = t_r + \frac{1}{\alpha} \ln \left( \frac{D + \alpha W e^{\alpha(t_d-t_r)}}{D} \right) \]

Which is a function of two continuous variable \( tr \) & \( T \).

**Optimality**

Our concern is to decide the ideal estimation of \( tr \) and \( T \) which limits \( TC_f (tr, T) \). The essential conditions for minimization of the complete cost work given by conditions (30) are

\[
\frac{\partial TC_f(t_r,T)}{\partial t_r} = 0, \quad \text{and} \quad \frac{\partial TC_f(t_r,T)}{\partial T} = 0 \quad \text{for} \quad i = 1, 2 \quad \text{which gives} \\
\frac{\partial TC_{f1}(t_r,T)}{\partial t_r} = \frac{D}{T} \left[ F t_d + c \beta e^{\beta(t_r-t_d)} + \frac{F}{\beta} \left( e^{\beta(t_r-t_d)} - 1 \right) + \left\{ c_1 - s(T - t_r) \right\} e^{-\delta(T-t_r)} - c_1 = 0 \right] \tag{31}
\]

\[
\frac{\partial TC_{f1}(t_r,T)}{\partial T} = -\frac{1}{T^2} \left\{ e^{\beta(t_r-t_d)} - e^{\beta(t_w-t_d)} \right\} \left\{ t_d + \frac{1}{\beta} \left( 1 - e^{\beta(t_d-t_r)} \right) \right\} + \left\{ \frac{1}{\beta} \left( e^{\beta(t_r-t_d)} - 1 \right) - (t_r - t_w) \right\} + \frac{D}{\alpha} \left\{ 1 - e^{\alpha(t_w-t_d)} \right\} + (t_w - t_d) + \frac{1}{\beta} \left( e^{\beta(t_r-t_d)} - e^{\beta(t_w-t_d)} \right) \left( 1 - e^{\beta(t_d-t_w)} \right) + \frac{1}{\beta} \left( e^{\beta(t_r-t_w)} - 1 \right) - (t_r - t_w)) \right\} + \frac{D}{T} \left\{ s(T - t_r) e^{-\delta(T-t_r)} + c_1 \right\} = 0 \tag{32}
\]

\[
\frac{\partial TC_{f2}(t_r,T)}{\partial t_r} = \frac{D}{T} \left[ F \left\{ e^{\beta(t_r-t_d)} t_d + \frac{1}{\beta} \left( e^{\beta(t_r-t_d)} - 1 \right) \right\} - \left\{ c_1 + s(T - t_r) \right\} e^{-\delta(T-t_r)} + c_1 + c \left( e^{\beta(t_r-t_d)} - 1 \right) \right] = 0 \tag{33}
\]

\[
\frac{\partial TC_{f2}(t_r,T)}{\partial T} = -\frac{1}{T^2} \left\{ A + \frac{H D t_d^2}{2} + F \left\{ \frac{1}{\beta} \left( e^{\beta(t_r-t_d)} - 1 \right) + (t_d - t_w) \right\} t_d + \frac{1}{\beta^2} \left( e^{\beta(t_r-t_d)} - 1 \right) - \frac{(t_d - t_w)^2}{2} \right\} \right\} + s \delta \frac{1}{\delta} \left\{ 1 - e^{-\delta(T-t_r)} \right\} + c_1 D \left\{ T - t_r - \frac{1}{\delta} \left( 1 - e^{-\delta(T-t_r)} \right) \right\} + c D \left\{ \frac{1}{\beta} \left( e^{\beta(t_r-t_d)} - 1 \right) - \left( (t_r - t_d) \right) \right\} + \frac{D}{T} \left\{ s(T - t_r) e^{-\delta(T-t_r)} + c_1 \right\} = 0 \tag{34}
\]
Case 1: When $t_d < t_w$

Case 2: When $t_d > t_w$

Figure 3: Convexity of cost function for FIFO w.r.t. $tr$ and $T$

Conditions [(31) & (32)] & [(33) & (34)] can be proved simultaneously for optimal values of $tri$ & $T_i$ (say $tri^*$ & $T_i^*$) for $i = 1, 2$ solved, it also clarified following sufficient equations

\[
\frac{\partial^2 TC_{fi}(tr, T)}{\partial tr^2} > 0, \quad \frac{\partial^2 TC_{fi}(tr, T)}{\partial T^2} > 0 \quad \text{and} \quad \left[\frac{\partial^2 TC_{fi}(tr, T)}{\partial tr^2} \times \frac{\partial^2 TC_{fi}(tr, T)}{\partial T^2} - \frac{\partial^2 TC_{fi}(tr, T)}{\partial tr \partial T}\right] > 0 \quad \text{for } i = 1, 2
\]

Details are solved in Appendix see Appendix1. Mathematically, it is very difficult to prove sufficient conditions, so convexities of cost function for both cases are shown graphically in graph3.

LIFO model formulation

In this paper we talk about a inventory framework embracing LIFO policy. In Last in first out (LIFO) approach the merchandise are put away in Owned Warehouse(OW) at first in the wake of satisfying the OW, remaining products are put away in Rented Warehouse(RW) however utilizes the products of RW before the products of OW to fulfill the interest so as to diminish the to decrease the stock conveying charge(holding cost). At first a ton size of QL units enters the framework. Subsequent to meeting the rainchecks, SL units enter the stock framework, out of which W units are stored in OW and the remaining (SL - W) units are stored in the RW. As the decay of thing is non-prompt, so at first, the units don't crumble for some period and after that the weakening starts. Extensively there can be two cases. Initially, when $td$ (time during which no weakening happens) is not exactly $tr$ (time during which stock in RW gets zero) and also when (time during which no crumbling happens) is more noteworthy than $tr$ (time during which the stock in RW gets zero).

Case 1: When $t_d < t_r$

During the time interval $[0, t_d]$ there is no deterioration so the inventory in RW is drained distinctly because of interest while in OW inventory level continues as before. Further, during the time interval $[t_d, t_r]$ the inventory level in RW is dropping to zero because of the joined impact of interest and crumbling and the inventory in OW gets exhausted because of decay alone. Presently, during the time interval $[tr, tw]$ consumption of inventory happens in OW because of the joined impact of interest and exhaust and tends to zero at time $tw$. Also, during the interval $[tw, t]$ the interest is calculated. The realization of the model in the time interval $[0,w]$ was graphically spoken to underneath in Figure 4.

Differential Eq. describing the level of stocks in RW & OW at time t for period (0, T).

\[
\frac{dI_2(t)}{dt} = -D \quad 0 \leq t \leq t_d
\]

\[
\frac{dI_2(t)}{dt} + \beta I_r(t) = -D \quad t_d \leq t \leq t_r
\]

\[
\frac{dI_0(t)}{dt} + \alpha I_0(t) = 0 \quad t_d \leq t \leq t_r
\]
\[ \frac{dI_r(t)}{dt} + \alpha I_0(t) = -D \quad t_r \leq t \leq t_w \] (38)

\[ \frac{dI_0(t)}{dt} = De^{-\delta(t-t)} \quad t_w \leq t \leq T \] (39)

The solutions of above five differential conditions (35), (36), (37), (38) & (39) with boundary conditions

\[ I_r(0) = S_L - W, I_r(t_r) = 0, \quad I_0(t_d) = W, \quad I_0(t_w) = 0 \quad \text{&} \quad B(t_w) = 0 \]

are

\[ I_r(t) = S_L - Dt - W \quad 0 \leq t \leq t_d \] (40)

\[ I_r(t) = \frac{D}{\beta} \left( e^{\beta(t_r-t)} - 1 \right) \quad t_d \leq t \leq t_r \] (41)

\[ I_0(t) = We^{\alpha(t_d-t)} \quad t_d \leq t \leq t_r \] (42)

\[ I_0(t) = \frac{D}{\alpha} \left( e^{\alpha(t_w-t)} - 1 \right) \quad t_r \leq t \leq t_w \] (43)

\[ B(t) = \frac{D}{\delta} \left\{ e^{-\delta(T-t)} - e^{-\delta(T-t_w)} \right\} \quad t_w \leq t \leq T \] (44)

The number of lost sales at time t is

\[ L(t) = D \left[ (t - t_w) - \frac{1}{\delta} \left( e^{-\delta(T-t)} - e^{-\delta(T-t_w)} \right) \right] \] (45)

Considering continuity of at \( I_r(t) \) at \( t=t_d \), it follows from eq. (40)& (41) that

\[ S_L - Dt_d - W = \frac{D}{\beta} \left( e^{\beta(t_r-t_d)} - 1 \right) \] (46)

This implies that maximum stock level per cycle is

\[ S_L = W + Dt_d + \frac{D}{\beta} \left( e^{\beta(t_r-t_d)} - 1 \right) \] (47)

Considering continuity of \( I_0(t) \) at \( t=t_r \) at it follows from equations (42) & (43) that

\[ We^{\alpha(t_d-t_r)} = \frac{D}{\alpha} \left( e^{\alpha(t_w-t_r)} - 1 \right) \] (48)

Putting \( t = T; \) in (44), is

\[ B(T) = \frac{D}{\delta} \left( 1 - e^{-\delta(T-t_w)} \right) \] (49)

Therefore, order quantity over replenishment cycle can be determined as

\[ Q_L = W + Dt_d + \frac{D}{\beta} \left( e^{\beta(t_r-t_d)} - 1 \right) + \frac{D}{\delta} \left( 1 - e^{-\delta(T-t_w)} \right) \] (50)

The total cost per cycle consists of following elements:

a) Ordering cost per cycle = A

b) The stock holding cost per cycle in

\[ RW = F \left\{ \frac{D}{\beta} \left( e^{\beta(t_r-t_d)} - 1 \right) \left( t_d + \frac{1}{\beta} \right) + \frac{Dt_d^2}{2} - \frac{D}{\beta} \left( t_r - t_d \right) \right\} \]

c) Inventory holding cost per cycle in

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\[ \text{OW} = H \left( \int_0^{t_d} W \, dt + \int_{t_d}^{t_r} I_0(t) \, dt + \int_{t_r}^{t_w} I_0(t) \, dt \right) \]

d) The backlogged cost per cycle is
\[
SC = s \frac{D}{\delta} \left[ \frac{1}{\delta} - \left( \frac{1}{\delta} + T - t_w \right) e^{-\delta(T-t_w)} \right]
\]
e) Opportunity cost due to lost sale per cycle is
\[
= c_1 D \left[ T - t_w - \frac{1}{\delta} \left( 1 - e^{-\delta(T-t_w)} \right) \right]
\]
f) Deterioration cost per cycle
\[
= c \left\{ W \left( 1 - e^{-\delta(t_d-t_r)} \right) + D(t_d - t_w) - D \left( \frac{1}{\alpha} \left( 1 - e^{\alpha(t_w-t_r)} \right) + \frac{1}{\beta} \left( 1 - e^{\beta(t_r-t_d)} \right) \right) \right\}
\]

Now, total relevant cost per unit time interval the cycle \((0, T)\)
\[
TC_{L1}(t_r, T) = \frac{1}{\tau} \left[ A + F \left( \frac{D}{\beta} \left( e^{\beta(t_r-t_d)} - 1 \right) \left( t_d + \frac{1}{\beta} \right) + \frac{D t_d^{\frac{3}{2}}}{2} \frac{D}{\beta} (t_r - t_d) \right) + H \left( W t_d + \frac{W}{\alpha} \left( 1 - e^{\alpha(t_d-t_r)} \right) \right) + \right.
\]
\[
\left. \frac{D}{\alpha} \left( t_r - t_w - \frac{1}{\alpha} \left( 1 - e^{-\delta(t_w-t_r)} \right) \right) \right] + \frac{D}{\delta} \left( \frac{1}{\delta} - \left( \frac{1}{\delta} + T - t_w \right) e^{-\delta(T-t_w)} \right) + c_1 D \left( T - t_w - \frac{1}{\delta} \left( 1 - e^{-\delta(T-t_w)} \right) \right) +
\]
\[
\left. c \left\{ W \left( 1 - e^{\alpha(t_d-t_r)} \right) + D(t_d - t_w) - D \left( \frac{1}{\alpha} \left( 1 - e^{\alpha(t_w-t_r)} \right) + \frac{1}{\beta} \left( 1 - e^{\beta(t_r-t_d)} \right) \right) \right\} \right] \quad (51)
\]

Case 2: When \(t_d > t_r\)

For this situation, time during which no disintegration happens is more noteworthy than the time during which stock in RW gets zero and the conduct of the model over the entire cycle \([0, T]\) has been graphically spoken to as in Figure 5.

**Figure 5:** Two-warehouse LIFO inventory system, when \(t_d > t_r\)

Differential eq. that describes the level of stock in RW & OW at time \(t\) during period \((0, T)\) is defined as:
\[
\frac{dI_r(t)}{dt} = -D \quad 0 \leq t \leq t_r \quad (52)
\]
\[
\frac{dI_0(t)}{dt} = -D \quad t_r \leq t \leq t_d \quad (53)
\]
\[
\frac{dI_0(t)}{dt} + \alpha I_0(t) = -D \quad t_d \leq t \leq t_w \quad (54)
\]
\[
\frac{dB(t)}{dt} = D e^{-\delta(T-t)} \quad t_w \leq t \leq T \quad (55)
\]
The solution of above four differential eq. (52), (53), (54) & (55) with conditions

\[ I_r(t_r) = 0, \quad I_0(t_r) = W, \quad I_0(t_w) = 0, B(t_w) = 0 \]

are

\[ I_r(t) = D(t_r - t) \quad 0 \leq t \leq t_r \quad (56) \]

\[ I_0(t) = W + Dt_r - Dt \quad t_r \leq t \leq t_d \quad (57) \]

\[ I_0(t) = \frac{D}{\alpha}(e^{\alpha(t_w - t)} - 1) \quad t_d \leq t \leq t_w \quad (58) \]

\[ B(t) = \frac{D}{\delta}(e^{-\delta(T - t)} - e^{-\delta(T - t_w)}) \quad t_w \leq t \leq T \quad (59) \]

Number of lost sales at time t is

\[ L(t) = D \left[ (t - t_w) - \frac{1}{\delta} \left( e^{-\delta(T - t)} - e^{-\delta(T - t_w)} \right) \right] \quad (60) \]

Considering continuity of \( I_0(t) \) at \( t = t_d \), it follows from equations (57) and (58) that

\[ W + Dt_r + Dt_d = \frac{D}{\alpha}(e^{\alpha(t_w - t_d)} - 1) \quad (61) \]

\[ t_w = t_d + \frac{1}{\alpha} \ln \left( 1 + \frac{aw}{D} + \alpha(t_r - t_d) \right) \quad (62) \]

Now, at \( t = 0 \) when \( I_r(t) = SL - W \) and solving equations (56)

\[ S_L = W + Dt_r \quad (63) \]

Putting \( t = T \) in equation (59);

\[ B(T) = \frac{D}{\delta}(1 - e^{-\delta(T - t_w)}) \quad (64) \]

Therefore, order quantity is

\[ Q_L = W + Dt_r + \frac{D}{\delta}(1 - e^{-\delta(T - t_w)}) \quad (65) \]

Total cost per cycle consists of following elements:

a) Ordering cost per cycle = A

b) Inventory holding cost per cycle in

\[ RW = F \left( \int_0^{t_r} I_r(t) dt \right) = \frac{FDt_r^2}{2} \]

c) Inventory holding cost per cycle in

\[ OW = H \left( \int_0^t W dt + \int_{t_r}^{t_d} I_0(t) dt + \int_{t_d}^{t_w} I_0(t) dt \right) \]

d) The backlogged cost per cycle is

\[ SC = s \frac{D}{\delta} \left[ \frac{1}{\delta} \left( \frac{1}{\delta} + T - t_w \right) e^{-\delta(T - t_w)} \right] \]
e) The opportunity cost per cycle due to lost sale is
\[ c_t \int_{t_w}^{T} (1 - e^{-\delta(T-t)})Dt \, dt \]

f) The deterioration cost per cycle is
\[ c_t \left( \frac{1}{\alpha} (e^{\alpha(t_w-t_d)} - 1) - (t_w - t_d) \right) \]

Now, total relevant cost per unit time during cycle \((0, T)\) is given by
\[
TC_{L2}(t_r, T) = \frac{1}{T} \left[ A + \frac{FDt_r^2}{2} + H \left( Wt_r + Dt_r(t_d - t_r) - \frac{D}{2}(t_d^2 - t_r^2) \right) + \frac{D}{\alpha} \left( \frac{1}{\alpha} (e^{\alpha(t_d-t_r)} - 1) - (t_w - t_d) \right) \right] + \frac{sD}{\delta} \left( \frac{1}{\delta} + T - t_w \right) e^{-\delta(T-t_w)} + c_t D \left( T - t_w - \frac{1}{\delta} (1 - e^{-\delta(T-t_w)}) \right) + c_t D \left( \frac{1}{\alpha} (e^{\alpha(t_w-t_d)} - 1) - (t_w - t_d) \right) 
\]

Therefore, total relevant cost per unit time during cycle \((0, T)\) is given by
\[
TC_L(t_r, T) = \begin{cases} 
TC_{L1}(t_r, T) & \text{if } t_d \leq t_r \\
TC_{L2}(t_r, T) & \text{if } t_d \leq t_r 
\end{cases} 
\]

Which is a function of two continuous variable \(tr\) and \(T\).

**Optimality:**

The necessary conditions for minimization of total cost function given by eq. (67) are
\[
\frac{\partial TC_{L1}(t_r, T)}{\partial t_r} = 0, \quad \text{and} \quad \frac{\partial TC_{L2}(t_r, T)}{\partial T} = 0
\]

\[
\frac{\partial TC_{L1}(t_r, T)}{\partial t_r} = \frac{1}{T} \left[ FD e^{\beta(t_r-t_d)} \left( t_d + \frac{1}{\beta} \right) + H \left( W e^{\alpha(t_d-t_r)} + \frac{D}{\alpha} \left( 1 - e^{\alpha(t_w-t_d)} \right) \left( 1 - X_1 \right) \right) + D \left( c_1 - \frac{2s}{\delta} - s(T - t_w) e^{-\delta(T-t_w)} - c_1 \right) X_1 + c \left( W e^{\alpha(t_d-t_r)} - DX_1 - D e^{\alpha(t_w-t_d)} (X_1 - 1) - e^{\beta(t_r-t_d)} \right) \right] = 0 
\]

\[
\frac{\partial TC_{L2}(t_r, T)}{\partial t_r} = -\frac{1}{T^2} \left[ A + F \left( \frac{D}{\beta} \left( e^{\beta(t_r-t_d)} - 1 \right) t_d + \frac{1}{\beta} \right) + \frac{D t_r^2}{2} - \frac{D}{\beta} (t_r - t_d) \right] + H \left( W t_r + \frac{D}{\alpha} \left( 1 - e^{\alpha(t_w-t_d)} \right) \right) + \frac{D}{\alpha} \left( t_r - t_w - \frac{1}{\alpha} \left( 1 - e^{\alpha(t_w-t_d)} \right) \right) + \frac{sD}{\delta} \left( \frac{1}{\delta} + T - t_w \right) e^{-\delta(T-t_w)} + c_1 D \left( T - t_w - \frac{1}{\delta} \left( 1 - e^{-\delta(T-t_w)} \right) \right) + c_1 e^{\delta(T-t_w)} - c_1 + c \left( e^{\alpha(t_w-t_d)} - 1 \right) X_2 \right] = 0 
\]

\[
\frac{\partial TC_{L2}(t_r, T)}{\partial T} = \frac{1}{T} \left[ FD t_r + H \left( W + D(t_d - 2t_r) + D t_r + \frac{D}{\alpha} \left( e^{\alpha(t_w-t_d)} - 1 \right) X_2 \right) + D \left( -s(T - t_w) e^{-\delta(T-t_w)} + c_1 \left( e^{\delta(T-t_w)} - 1 \right) + c \left( e^{\alpha(t_w-t_d)} - 1 \right) X_2 \right) \right] = 0 
\]

\[
\frac{\partial TC_{L2}(t_r, T)}{\partial T} = \frac{1}{T} \left[ sD(T - t_w) e^{-\delta(T-t_w)} + c_1 D \left( 1 - e^{-\delta(T-t_w)} \right) - CD \right] - \frac{1}{T^2} \left[ A + \frac{FDt_r^2}{2} + H \left( W t_r + Dt_r(t_d - t_r) - \frac{D}{2}(t_d^2 - t_r^2) \right) + \frac{D}{\alpha} \left( \frac{1}{\alpha} (e^{\alpha(t_w-t_d)} - 1) - (t_w - t_d) \right) \right] + \frac{sD}{\delta} \left( \frac{1}{\delta} + T - t_w \right) e^{-\delta(T-t_w)} + c_1 D \left( T - t_w - \frac{1}{\delta} \left( 1 - e^{-\delta(T-t_w)} \right) \right) + cD \left( \frac{1}{\alpha} (e^{\alpha(t_w-t_d)} - 1) - (t_w - t_d) \right) = 0 
\]
Eq. [(68) & (69)]; [(70) & (71)] can be derived simultaneously for optimal values of \( tri \) & \( Ti \) (say \( tri^* \) and \( Ti^* \)) for \( i=1,2 \) provided,

\[
\frac{\partial^2 T_{CLi}(tr,T)}{\partial t_i^2} > 0, \frac{\partial^2 T_{CLi}(tr,T)}{\partial T^2} > 0
\]

And \[
\left[ \frac{\partial^2 T_{CLi}}{\partial t_i^2} \times \frac{\partial^2 T_{CLi}}{\partial T^2} \right] - \left[ \frac{\partial^2 T_{CLi}}{\partial T \partial t_i} \times \frac{\partial^2 T_{CLi}}{\partial t_i \partial T} \right] > 0 \text{ for } i = 1, 2
\]

Details are provided in Appendix see Appendix 2.

Mathematically, it is very difficult to prove the sufficient conditions, so convexities of cost function for both cases are shown graphically in figure 6.

**Case 1:** When \( td < tr \), **Case 2:** When \( td \geq tr \).

**Figure 6:** Convexity of cost function for LIFO w.r.t. \( tr \) and \( T \)

**V. NUMERICAL AND SENSITIVITY ANALYSIS**

To delineate outcomes, let us consider a stock framework with accompanying information: \( A=\$\ 250/\text{per cycle}, c=\$\ 10/\text{unit}, s = \$ \ 5/\text{unit/year}, c1 = \$ \ 5/\text{unit/year}, H = \$ \ 0.5/\text{unit}, F = \$ \ 0.7/\text{unit}, W = 200\text{ units}, D=300\text{ units}, \alpha = 0.05, \beta = 0.03\text{units/year}. Utilizing proposed arrangement system outcomes are as per following:

**For FIFO Model**

\( tw= 0.658\text{ year}, \ tr = 1.206\text{ year}, T = 1.345\text{ year}, SF = 368.496\text{ units}, QF = 407.748\text{ units}, TCF = \$ 360.854\text{ and} \text{deterioration cost}=\$ 67.858\)

**For LIFO Model,**

\( tw= 1.197\text{ year}, \ tr = 0.559\text{ year}, T = 1.337\text{ year}, SL = 366.072\text{ units},QL = 409.413\text{ units}, TCF = \$ 365.648\text{ and} \text{deterioration cost}=\$ 88.505\)

As the all out expense in FIFO strategy is not exactly of LIFO strategy, along these lines FIFO dispatching strategy is favored over LIFO. Further, the affectability examination on significant boundaries \( td, \alpha, \beta, \delta \) and holding costs (H and F) have been talked about and appeared in table 1-5 individually. So as to consider the impact of non-crumbling period for example \( td \) on the arrangement, we consider the various estimations of \( td \) and results are summed up in Table 1:

**Table 1:** Effect of non-deteriorating period (\( td \)) on policy selection

<table>
<thead>
<tr>
<th>td</th>
<th>FIFO</th>
<th>LIFO</th>
<th>Policy Selected</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tw</td>
<td>tr</td>
<td>T</td>
</tr>
<tr>
<td>0</td>
<td>0.6</td>
<td>1.2</td>
<td>1.3</td>
</tr>
<tr>
<td>0.0</td>
<td>0.6</td>
<td>1.2</td>
<td>1.3</td>
</tr>
<tr>
<td>0.5</td>
<td>0.6</td>
<td>1.2</td>
<td>1.3</td>
</tr>
<tr>
<td>0.7</td>
<td>0.6</td>
<td>1.2</td>
<td>1.3</td>
</tr>
</tbody>
</table>

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Following observations can be made from Table 1:

As td (time during which no disintegration happens) builds, cycle length (T) and cost of falling apart units diminishes which in the long run outcomes in a lessening in the all out normal expense.

When td = 0 for example weakening beginnings toward the start of the cycle (prompt crumbling) the decay cost increments on account of immediate breaking down things when contrasted with the non-momentary things as the quantity of disintegrating units are more. Thus non quick decaying units are a lot of valuable in lessening the disintegration cost as it decreases the Total expense. Likewise the Deterioration cost is higher in LIFO strategy than that of FIFO strategy, so clearly the chief will embrace FIFO strategy as opposed to LIFO strategy.

So as to study the effect of decay in defining the approach, the maintenance cost of both storage facilities are considered equivalent (H = F = 0.5) and the proposed model has been shown under two circumstances. Right off the bat it is expected that the disintegration rate in OW is more prominent than that of the RW and the other way around.

Table 2: Effect of deterioration rates on policy selection

<table>
<thead>
<tr>
<th>r</th>
<th>QF</th>
<th>TCF</th>
<th>QL</th>
<th>TCL</th>
<th>Policy suggested</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>438.746</td>
<td>317.837</td>
<td>470.696</td>
<td>290.642</td>
<td>LIFO</td>
</tr>
<tr>
<td>0.5</td>
<td>445.287</td>
<td>322.186</td>
<td>462.957</td>
<td>307.252</td>
<td>LIFO</td>
</tr>
<tr>
<td>1</td>
<td>453.271</td>
<td>327.488</td>
<td>453.271</td>
<td>327.488</td>
<td>EITHER</td>
</tr>
<tr>
<td>2</td>
<td>468.642</td>
<td>337.671</td>
<td>433.352</td>
<td>366.243</td>
<td>FIFO</td>
</tr>
</tbody>
</table>

Based on the outcome as appeared in Tables 2, we get following experiences:

At that point when weakening rate in OW is higher than that of RW i.e. TCF > TCL thus it is ideal to clear OW prior to RW as to keep up freshness of goods which bring about lesser deterioration cost. Consequently in that conditions leader would incline toward FIFO dispatch policy than LIFO. When TCF = TCL both the strategies submits indistinguishable outcomes, henceforth the chief may utilize either FIFO or LIFO dispatch strategy. RW disintegrate all the more quickly so keeping stock in RW for longer period would bring about more weakening expense. Thus the chief would utilize LIFO dispatch strategy instead of FIFO. So as to examine the impact of holding cost on the approach choice, by taking various blends of H and F, and the proposed model has been exhibited under two circumstances. Initially it is accepted that weakening rate in OW is more noteworthy than that of the RW and the other way around.

Table 3: Effect of maintenance costs on policy selection (When deterioration rate in OW is higher)

<table>
<thead>
<tr>
<th>H</th>
<th>F</th>
<th>TCF</th>
<th>TCL</th>
<th>Policy Suggested</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>334.336</td>
<td>353.576</td>
<td>FIFO</td>
</tr>
<tr>
<td>0.5</td>
<td>0.6</td>
<td>348.490</td>
<td>357.902</td>
<td>FIFO</td>
</tr>
<tr>
<td>0.5</td>
<td>0.7</td>
<td>360.854</td>
<td>357.648</td>
<td>LIFO</td>
</tr>
<tr>
<td>0.5</td>
<td>0.6</td>
<td>338.622</td>
<td>366.916</td>
<td>FIFO</td>
</tr>
<tr>
<td>0.7</td>
<td>0.6</td>
<td>353.063</td>
<td>370.967</td>
<td>FIFO</td>
</tr>
<tr>
<td>0.5</td>
<td>0.7</td>
<td>365.706</td>
<td>362.467</td>
<td>LIFO</td>
</tr>
<tr>
<td>0.5</td>
<td>0.7</td>
<td>342.856</td>
<td>380.094</td>
<td>FIFO</td>
</tr>
</tbody>
</table>

Table 4: Effect of holding costs on policy selection (When deterioration rate in RW is higher)

<table>
<thead>
<tr>
<th>H</th>
<th>F</th>
<th>TCF</th>
<th>TCL</th>
<th>Policy Suggested</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>370.305</td>
<td>362.393</td>
<td>LIFO</td>
</tr>
<tr>
<td>0.5</td>
<td>0.6</td>
<td>379.830</td>
<td>365.417</td>
<td>LIFO</td>
</tr>
<tr>
<td>0.7</td>
<td>0.5</td>
<td>388.076</td>
<td>368.115</td>
<td>LIFO</td>
</tr>
<tr>
<td>0.7</td>
<td>0.5</td>
<td>375.431</td>
<td>376.060</td>
<td>FIFO</td>
</tr>
<tr>
<td>0.6</td>
<td>0.6</td>
<td>385.220</td>
<td>380.882</td>
<td>LIFO</td>
</tr>
<tr>
<td>0.7</td>
<td>0.7</td>
<td>393.726</td>
<td>382.395</td>
<td>LIFO</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>380.491</td>
<td>387.566</td>
<td>FIFO</td>
</tr>
</tbody>
</table>
The computational outcomes as appeared in Tables 3 & 4, we get accompanying administrative bits of knowledge:

Also, if the holding cost in RW is more notable than that of OW however decay rate in RW is not as strong as OW, at that point it is unambiguous from the outcomes that expense related with LIFO dispatching strategy is not exactly the FIFO dispatching strategy. Thus, LIFO strategy is liked.

When \( TC_F = TC_L \), both policies give the same results, hence decision maker can use the FIFO or LIFO dispatch policy. Now one studies effect of backlogging parameter \( \delta \) on the choice of policy. The sensitivity analysis is performed by changing backlogging parameter \( \delta \) & keeping all other parameters as same.

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>( S_f )</th>
<th>( Q_f )</th>
<th>( TCF )</th>
<th>( SL )</th>
<th>( QL )</th>
<th>( TCL )</th>
<th>Policy Suggests</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>370.87</td>
<td>4058.01</td>
<td>363.24</td>
<td>370.48</td>
<td>404.69</td>
<td>364.06</td>
<td>FIFO</td>
</tr>
<tr>
<td>0</td>
<td>368.49</td>
<td>407.746</td>
<td>360.85</td>
<td>368.07</td>
<td>407.41</td>
<td>361.64</td>
<td>FIFO</td>
</tr>
<tr>
<td>0</td>
<td>365.37</td>
<td>411.457</td>
<td>357.72</td>
<td>364.91</td>
<td>411.10</td>
<td>358.48</td>
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<tr>
<td>0</td>
<td>361.12</td>
<td>416.715</td>
<td>353.47</td>
<td>360.62</td>
<td>416.33</td>
<td>354.19</td>
<td>FIFO</td>
</tr>
</tbody>
</table>

It is seen from table 5 that with the diminishing in multiplying boundary \( \delta \), there is a decline underlying stock & complete normal expense. Since expanding multiplying rate infers a greater amount of accumulated interest. So it is fitting when multiplying rate is more, association should arrange bigger amount so as to fulfill the accumulated interest. As the \( TC_F \geq TC_L \), thus, in this case FIFO policy is suggested.

VI. CONCLUSION:

Generally significant for directors that manage transient items utilizing FIFO, instead of LIFO, is a typical acknowledged act of ensuring that the items are dispatched at its greatest newness. In present paper, a two-distribution inventory model for falling apart stock things with FIFO dispatching strategy has been proposed. Numerous analysts expected that in the two distribution inventory stock model interest rate is consistent however it isn't sensible in a large portion of its interest rate is time subordinate. Along these lines in this contemplated model we consider straight time subordinate interest rate. Consequently this investigation is a lot of appropriate for some pragmatic business circumstances. In this paper, we talked about a two stockroom stock model with the FIFO dispatching strategy and direct interest for disintegrating things and furthermore altered the two distribution centre stock model. Numerical articulations for different boundaries are gotten.

REFERENCES